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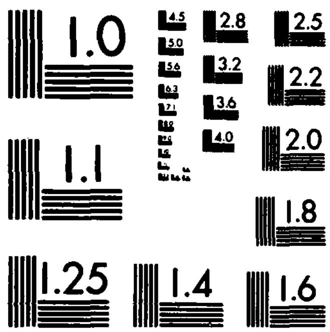

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DEVELOPMENT AND APPLICATION OF OPTIMIZATION TECHNIQUES FOR COMPOSITE LAMINATES

MASTER'S THESIS

AFIT/GAE/AA/83S-4

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SCHOOL OF ENGINEERING

WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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DEVELOPEMENT AND APPLICATION
OF OPTIMIZATION TECHNQUES FOR
COMPOSITE LAMINATES

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technolgy

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Requirements for the Degree of

Master of Science

by

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The willingness of my faculty advisor, Dr. A. N. Palazotto, to trust my judgement and accept this project as thesis material is also appreciated.

Table of Contents

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

Matrices

A - Inplane stiffness

a* - Inverse of A times total thickness

Ao - Inplane stiffness at current position in design space

$\dot{A}z$ - Change in $|A|$ for scalar move along vector Z

F - Quadratic strength parameters in stress space

G - Quadratic strength parameters in strain space

Q - Lamina elastic modulus

T - Coefficient matrix defined by equation (48)

Vectors

F - Linear coefficients of quadratic strength criterion in stress space

G - Linear coefficients of quadratic strength criterion in strain space

h - Ply group thicknesses

N - Loads in terms of stress resultants

n - Unit normal to constant thickness plane

W - Negative of sum of gradient vectors to active constraints

Z - Search direction

ϵ - Strain

σ - Stress

Scalars

a - components of the inverted A matrix

b - Strain sphere radius

C - Value of the failure constraint equation for ply P and load L

e_1 - numerical offset to ensure design stays slightl[illegible] feasible region

e_2 - defines how close to zero constraint must be in order to be active

Ex - Longitudinal Young's modulus

Ey - Transverse Young's modulus

m - number of ply groups

n - number of independent loads

n - Total number of constraints

P,Q,R - Intermediate results given by equation (39)

R - Strength ratio

r - distance to origin in design space

S1,S2 - Bounds on bisection search

Smax- distance to first h =0 constraint

S - Shear strength

U - Strain energy density

U ,U - Transformation invarients

X - Longitudinal tensile strength

X' - Longitudinal compressive strength

Y - Transverse tensile strength

Y' - Transverse compressive strength

θ - Ply orientation

λ - N /N or constant in Langrange equation

ν_x - Longitudinal Poisson's ratio

ν_y - Transverse Poisson's ratio

σ - Variance

Ψ - Laminate rigid body rotation

Subscripts

,h - Partial derivative with respect to ply group thickness

i,j,k - Tensor indices

L - Realated to independent load vector L

x,y,s - Ply axis system

1,2,6 - Laminate axis system

,θ - Partial derivative with respect to ply orientation

Superscripts

(P),(k),(i) - Transformed from orientation of ply group P, k, or i

o - Reference parameter

' - second independent load

ABSTRACT

The design of a composite panel requires some way of finding the minimum thickness laminate which will withstand the load requirements without failure. The mathematical complexity of this problem dictates the use of non-linear optimization techniques. Although there are sophisticated optimization programs available capable of solving for the ply ratios, these programs are not often used in preliminary design because they require a large computer and some knowledge of the program's operation. As an alternative, specialized laminate optimization programs were developed which are compact and efficient enough to run on microcomputers. Only stresses at a point and inplane loads and deflections are considered. The programs are simple to use and require no knowledge of optimization. Techniques are developed in this thesis that find minimum thickness laminates with either ply ratios or ply angles as design variables. In addition, a method is presented for finding the optimimum orientation for the axis of symmetry of an orthotropic laminate. The orthotropic laminate program uses an approximate failure theory, as suggested by Tsai, that has been found to speed computations dramatically.

Many test cases were run with these programs to demonstrate the weight savings possible over quasi-isotropic laminates. Of particular interest is performance of the laminates under multiple independent loads. Initial orientations for the programs to operate on were studied, and 0/90/45/-45 laminates were found to be an effective starting point for design.

The approximate failure criterion made analytic investigations of optimized laminates possible. A method of plotting maximum strain

energy density as a function of the shear-stress-free laminate orientation is derived t demonstrate how the laminates adapt to multiple design load requirements in the optimization process. Also, an optimality criterion is derived which is satisfied by each ply group at the minimum thickness condition.

I. INTRODUCTION

Background

Almost any introductory text on composite materials will make a statement to the effect that one of the principle advantages of composites is the possibility of tailoring the laminate to suit the structural requirements. By using the directional nature of the material to advantage, highly efficient structures should be possible. Yet, except for uniaxial loads, no suggestion is made for selecting these tailored laminates. The omission is not accidental, but is due to the difficulity of converting the equations for laminate analysis into equations for laminate design.

When sizing an isotropic plate, the orientations and the number of plies at each orientation can be variable. Although analysis equations for finding the response of a given laminate are well known, these equations cannot be solved to yield the best laminate for a given set of requirements. Besides being non-linear, structural design requirements, such as strength, are stated as inequalities. There is no way to know how to assign equalities to the equations and solve for the design variables. We cannot tell which combination of requirements will be "critical" for the best design.

A common approach to sizing laminates is to asume the plies are acting independently. For strength requirements, this is referred to as netting analysis. Although in general this is a bad approximation, reasonable results can be obtained for 0/90 laminates with no shear. With any other case, such as additional ply groups, off-axis loads, or

multiple independent loads, netting analysis cannot provide a reasonable basis for design. The plies within a laminate interact in a complex manner and cannot be considered independent. Because of the interaction, there are no simple formulas for proper sizing, nor is intuition a reliable guide.

Non-linear optimization techniques developed over the last 20 years provide a sound mathematical basis for laminate sizing. The techniques should not be thought of as the final step in design, used to shave off a couple percent of weight, but as the starting point of design. Optimization can be applied to any design constraint that can be mathematically modelled. Constraints may include stiffness, strength, stability, minimum gage, and dynamic response. In this thesis, the author has chosen to work only with strength constraints. Besides being an essential element of design, it is one of the few constraints which can be described as a point problem, assuming loads do not change as the laminate changes. The optimal laminate will be considered as one with minimum thickness, and thus weight. For constraints such as stability, the optimizer must be coupled to a structual analysis method, such as finite elements, in order to describe the geometry and boundary condition influences. The assumption that optimization for strength can be dealt with as a point problem is completely valid only for a determinate structure. The optimization procedure will have to be coupled to a structural analysis code in some iterative process in order to properly size indeterminate structural elements. Still, the simple methods and programs presented here should be of aid in much of the initial design process.

The role of optimization is particularly inportant when designing for multiple loading conditions. A wing panel must sustain several

different flight conditions, as well as ground loads. Not only are the magnitudes of these loads changing with time, but the orientation of the load principle axes may also change. For directional materials, it will often be convenient to think in terms of shear-free loads and an angle that transforms the loads to the laminate axis system. Because of the laminate's anisotropic strength, changes in the principle axis leads to a problem that does not exist for isotropic materials; it is impossible to pick a severest load condition by inspection and size the laminate to that load alone. In fact, there may not be any single severest condition. For a minimum weight laminate, some of the plies may be near failure for one load, while other plies are critical for a different load. One result of this added complication is that optimization results cannot be tabulated in a design manual. There is no way to characterize all the possible loading combinations into a finite set of graphs. Instead, the computer must become an integral part of preliminary design.

If optimization is so valuable to the design of composite laminates, why isn't it in common useage? After all, the basic methods of non-linear optimization are well developed and can handle much more complex problems than sizing a laminate. Indeed, laminate sizing is a comparatively well behaved problem, with typically only a few design variables and constraints. Part of the answer may be the reluctance to use a complex and general code requiring a main-frame computer. In addition, there may be some lack of confidence in the procedure. This thesis presents some specialized, user-friendly codes which can be run on microcomputers at the designer's desk. Hopefully, by having a desk-top computer that only requires the user to respond to some simple prompts for input, further application of optimization will be

encouraged.

The potential for applying optimization techniques to composites has not escaped the attention of other authors. At least 2 programs exist in a documented, publicly available form. One is COMAND by Vanderplaats [1] which couples a laminate analysis program to an existing general optimization code, also by the same author. Maximum strain failure criteria are used, and minimum values of the A matrix components can be entered to account for minimum stiffness requirements. Another program was written by Khot [2]. Instead of a direct numerical optimization, this program relies on the assumption that strain energy density will be equal for all ply groups as the laminate approaches minimum thickness [3]. An iterative procedure for adjusting the number of plies is derived from the assumed optimality condition. The program also includes an approximate buckling constraint, based on "smeared" laminate properties. The optimization routines are coupled to a finite element code to update the stress state as the composite panels change. Neither of these programs meets the requirement for simplicity of use which is the goal of this thesis.

Without a numerical optimization program, the minimum thickness laminates can still be studied if there is only one free variable, such as the best angle in an angle-ply laminate. Some of these one-dimensional searches are presented in [4]. This reference is notable because it includes the approximate, strain-sphere failure criterion discussed later in this thesis.

The programs written in the course of this work are all in BASIC. The particular computers were chosen somewhat arbitrarily, but the codes should be readily transferable to other computers with a minimum of change. Optimization with the quadratic failure criteria with a

complete set of laminate property outputs requires about 12 kilo-bytes of memory. The angle optimization can be attached for about 2k more memory. A simplified version based on an approximate failure criteria fits in less than 6K. Programs have been written for the Timex-Sinclair 1000 [5], the Epson HX-20 [6], and the Texas Instruments CC-40 . These last 2 microcomputers were picked because they offer true desk-top capability; the original goal of the project.

Laminate Theory

The developement of the laminate plate theory equations will follow Tsai and Hahn [7] wherever possible. The difference will be that vector notation is used more extensively in this thesis. The plates will be subject only to inplane loads and deflections. The order of plies in the laminate, or stacking sequence, is not a factor in the optimization procedure. However, for the inplane deflection restriction to be valid, the actual laminate would have to be symmetric. That is, for any ply at orientation θ, a distance Z above the midplane, there is a corresponding ply of the same orientation at minus Z. For these restrictions, strain is a constant through the thickness and the stress-strain relation is simply

$$\vec{N} = [A]\vec{\varepsilon} \tag{1}$$

where

$$A_{jk} = \sum_{i=1}^{m} Q_{jk}^{(i)} h_i \tag{2}$$

$\vec{\varepsilon}$-laminate strain vector

$\vec{N}$-load vector in terms of stress resultants

$Q_{jk}^{(i)}$-modulus component transformed from the orientation of the i'th ply group

m-number of ply groups

h_i-thickness of the i'th ply group

Several ways exist to perform the transformations. The programs listed in Appendices B-D use an invariant formulation with multiple-angle functions as given in reference [7]. In terms of engineering constants, the Q's are given by

$$Q_{xx}=mE_x \qquad Q_{yy}=mE_y$$
$$Q_{xy}=m\nu_y E_x \qquad Q_{ss}=E_s \tag{3}$$

where

$$m = (1-\nu_x\nu_y)^{-1}$$

Ex is the longitudinal Young's modulus, νx the longitudinal Poisson's ratio, Ey the transverse Young's modulus, and νy the transverse Poisson's ratio.

The axis system convention is shown in Figure 1. x, y, and s subscripts denote properties in the ply axis system, and 1, 2, and 6 denote properties in the laminate axis system.

A ply group will be defined as all the plies of a particular orientation and material (for hybrids), whether or not they are actually adjacent in the laminate. In the optimization procedure, ply group thickness is handled as a continous variable. The individual ply as a discret unit is ignored. After the procedure is finished, we must divide the ply group thickness by the thickness of an individual ply and round-off to get the integer mumber of plies required. A logical way of rounding-off must be a topic of future research. For now, rounding-up should be assumed for all ply groups. The term "ply ratio" will also be used. This is the ratio of a particular ply group thickness to the total laminate thickness.

For the graphs and tables presented in this thesis, the conventional lamination code becomes awkward. Instead, the notation

$$(0/90/\pm45)$$

refers to the class of laminates with those orientations, with ply group thickness determined by the optimization procedure. Also,

$$(0_1/90_1/\pm45_1)$$

refers to a laminate with the stated orientations and equal ply ratios,

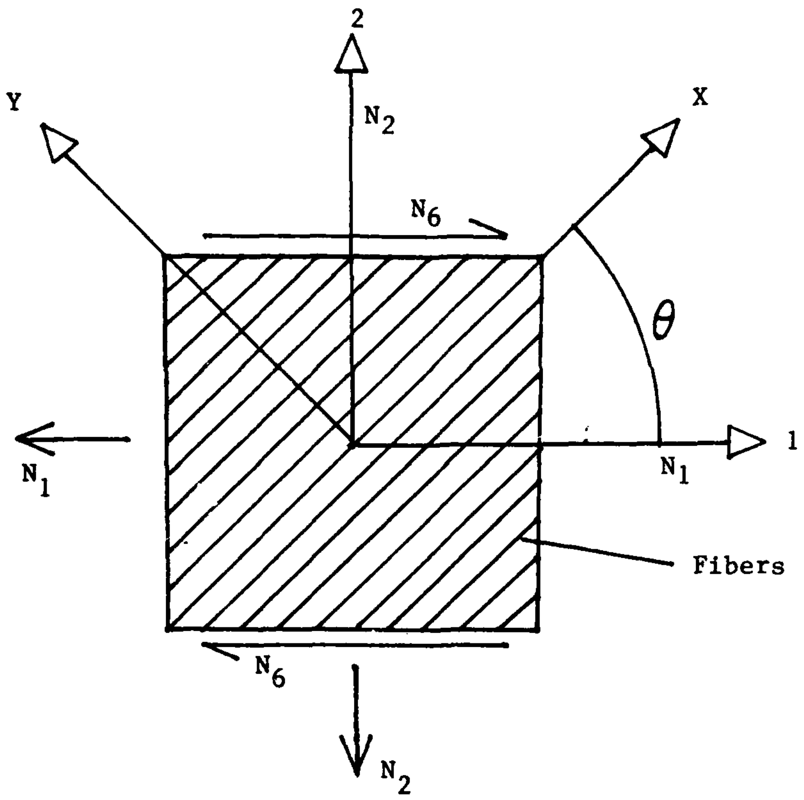


FIGURE 1: Laminate and Ply Axis Systems

where no optimization has been performed. Total thickness is still a continuous variable.

Failure Criteria

One of several failure criteria could be selected for incorporation in the optimization procedure [8]. The quadratic tensor polynomial or Tsai-Wu criterion was selected because it fits experimental data well [7] and because it reduces the number of constraints as compared to maximum stress or strain criteria. The quadratic failure criterion is based on fitting an ellipse to the experimental failure strengths of a unidirectional lamina. The form of the equation accounts for interaction between the stresses causing failure. As in most laminate failure criteria, each ply in the laminate must be interrogated seperately in order to determine if failure has occured. In this thesis, first-ply failure is adopted, in contrast to a progressive failure model.

The quadratic failure criterion takes the form

$$F_{ij}\sigma_i\sigma_j + F_i\sigma_i - 1 \leq 0 \qquad i,j = 1,2,6 \tag{4}$$

The F's are related to experimental data as follows

$$\begin{aligned} F_{xx} &= \frac{1}{XX'} \\ F_x &= \frac{1}{X} - \frac{1}{X'} \\ F_{yy} &= \frac{1}{YY'} \\ F_y &= \frac{1}{Y} - \frac{1}{Y'} \\ F_{xy} &= F^*_{xy}\sqrt{F_{xx}F_{yy}} \end{aligned} \tag{5}$$

where X -longitudinal tensile strength

X' -longitudinal compression strength

Y -transverse tensile strength

Y' -transverse compression strength

S -shear strength

F^*_{xy} -non-dimensional interaction term

F^*_{xy} has not yet been accurately measured since it requires a reliable biaxial stress test. From geometric bounds and by analogy to isotropic materials (Von Mises failure theory) a value of -1/2 is usually taken, and is used throughout this thesis.

Stating the failure criteria in terms of strain is convenient. In strain space the failure envelopes stay fixed even if the ply ratios of the laminate are changed. The strain limits of a ply are independent of the laminate stiffness. This is an important conceptual simplification when ply ratios are variable. The failure criterion can be rewritten as

$$G_{ij}\varepsilon_i\varepsilon_j + G_i\varepsilon_i - 1 \quad 0 \qquad\qquad i,j = 1,2,6 \tag{6}$$

where the G's are found by applying the stress-strain relations, assuming linear elasticity to failure. Then

$$\begin{aligned} G_{k\ell} &= F_{ij}Q_{ik}Q_{j\ell} \\ G_j &= F_iQ_{ij} \end{aligned} \qquad\qquad i,j,k,\ell = 1,2,6 \tag{7}$$

The G and F matrices can be transformed for off-axis plies by a second-order tensor transformation, just as with the elasticity components. The linear terms of the equation (G vector) are transformed by

$$\begin{aligned} G_1 &= P + q\cos 2\theta \\ G_2 &= P - q\cos 2\theta \\ G_6 &= q\sin 2\theta \end{aligned} \tag{8}$$

$$P = \frac{1}{2}(G_x + G_y) \; : \quad q = \frac{1}{2}(G_x - G_y)$$

Figure 2 shows the failure enveloped for a 0/90 laminate of T300/5208 (Graphite/Epoxy). The envelopes are actually three-dimensional, and shear strain is not shown. Only the region enclosed by both ellipsoids is considered safe.

An approximate first-ply failure envelope was suggested in reference [7]. The envelope is based on recognizing there is a first-ply failure domain common to all possible ply orientations, and thus independent of the orientation of any particular ply. Figure 3 shows failure envelopes for several orientations. There is an inner envelope defined by the $0^o$and 90^oplies, within which no failure occurs for any possible orientation. Note that $0^o$and 90^oplies do not always define this space for other material systems. By using this inner envelope, we have a failure criterion which applies to the laminate as a whole, and does not need to be interrogated on a ply-by-ply basis. It is convenient to fit some analytic surface into the envelope. Since tension loads are of primary interest in this work (because there are no stability constraints), a sphere centered on the origin was selected to give a conservative approximation of the inner envelope. The approximate failure criterion can then be written

$$\varepsilon_1^2 + \varepsilon_2^2 + \frac{1}{2}\varepsilon_6^2 \leq b^2 \qquad (9)$$

The sphere's radius, b, can be set equal to the minimum lamina strain, taken directly from experimental data. The criterion will be referred to as a maximum strain-sphere.

The strain-sphere criterion will not be acceptable for uniaxial laminates, or for loads in the 3rd quadrant (compression-compression),

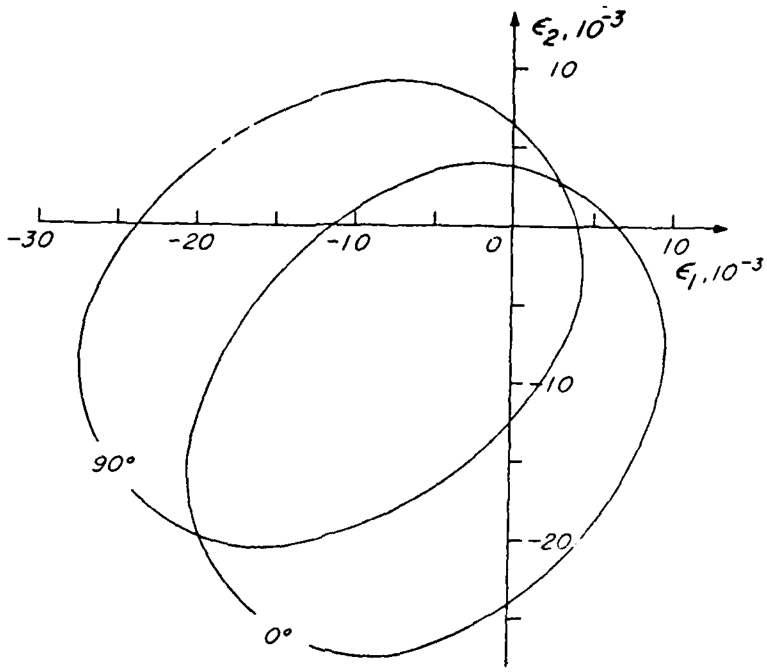


FIGURE 2: Quadratic Criterion Failure Envelopes

Material: T300/5208

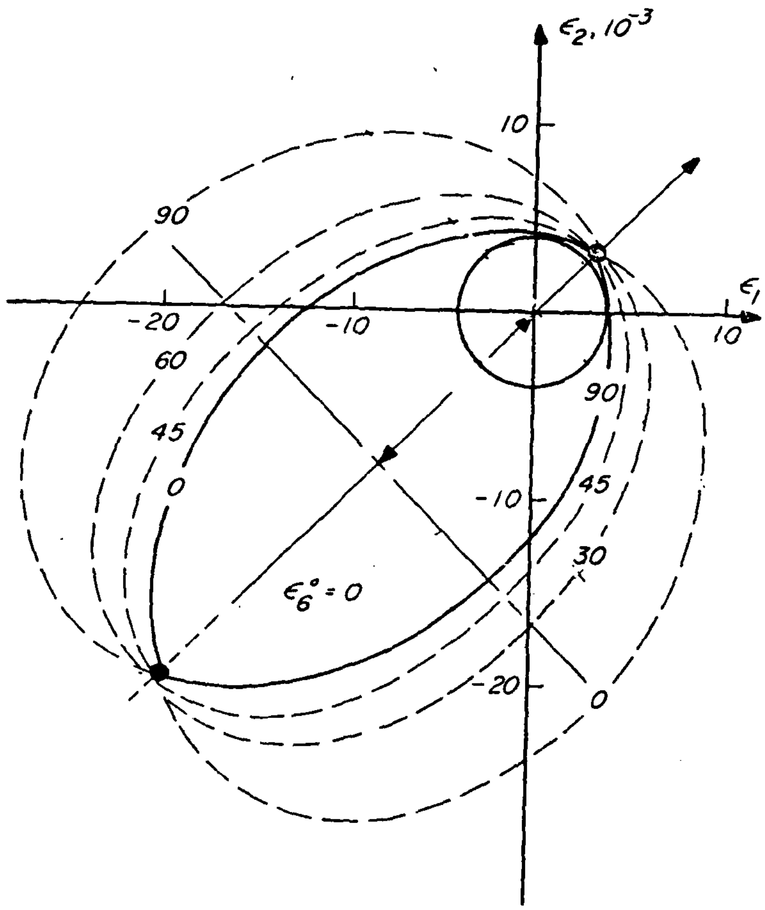


FIGURE 3: First Ply Failure Inner Envelope with Inscribed Sphere

Material: T300/5208

but otherwise is of some value. The simplicity of the criterion more than doubles the speed of the optimization algorithm. For optimization with tension-tension loads, it has been found to be about 7% conservative, as compared to the Tsai-Wu criterion. Thus, for quick answers, the approximation is adequate. In addition to allowing for extra fast computation, the maximum strain-sphere is simple enough to allow analytic investigations of the optimization process, as will be discussed in later sections.

Failure Constraints in Design Space

Normally, failure envelopes show the set of loads (or strains) that can be sustained by a particular laminate. For design purposes, the set of laminates that can sustain particular loads would be more desirable. Instead of stress or strain coordinates on a graph, the coordinates should be the design variables, for example, ply group thicknesses. Unfortunately, there may be an arbitrary number of design variables, and therefore dimensions to the problem. Therefore, general design graphs cannot actually be drawn, but the concept is important to understanding the optimization process.

One way of showing the set of laminates that could sustain a given combination of loads is to make a plot which divides design space into two regions; a region where the laminates would not fail for any of the given loads (called the feasible region), and a region where the laminates would fail (called the infeasible region). Any point in design space defines a unique laminate. We will restrict the discussion to taking ply group thicknesses as the only design variables. The boundary between the feasible and infeasible regions is the surface defined by the the failure criteria equations when made into an equality and plotted as functions of the thicknesses. With the quadratic failure criterion, we can write

$$\vec{\varepsilon}^{\,T}_{(L)} \; |G^{(P)}| \; \vec{\varepsilon}_{(L)} + \vec{G}^{(P)^T} \; \vec{\varepsilon}_{(L)} = 1 \tag{10}$$

where the subscript L designates the strains associated with a particular set of loads and superscript P denotes a transformation from a particular angle. Equation (10) can be shown to be a function of the h's (ply group thickness) by substituting

$$\vec{\epsilon}_{(L)} = |A^{-1}| \vec{N}_{(L)} \tag{11}$$

An important feature of working in design space is that the constraint surfaces for more than one set of loads can be plotted together. The final result is several surfaces in the design space, with the outermost surfaces forming the boundary between feasible and infeasible space (Figure 4).

If there are only two design variables, we can actually draw these design graphs. Figures 5, 6, and 7 are plots of the constraint curves for a 0/90 laminate under a single biaxial load. The three figures are for three different failure criteria. To define the feasible region, the maximum strain criterion requires the number of surfaces to be three times the number of ply groups times the number of independent loads (only 4 curves are shown in Figure 5 because shear strain is zero for the particular class of laminate and the given load). The quadratic criterion requires the number of surfaces to be equal to the number of ply groups times the number of loads. Reducing the number of constraints speeds the optimization procedure. Speed of operation is another motivation for choosing the quadratic criteria for the majority of additional work. The approximate strain-sphere criterion is simplier yet, with only a single surface for each independent load. Because it is a conservative approximation, only limited use will be made of this criterion.

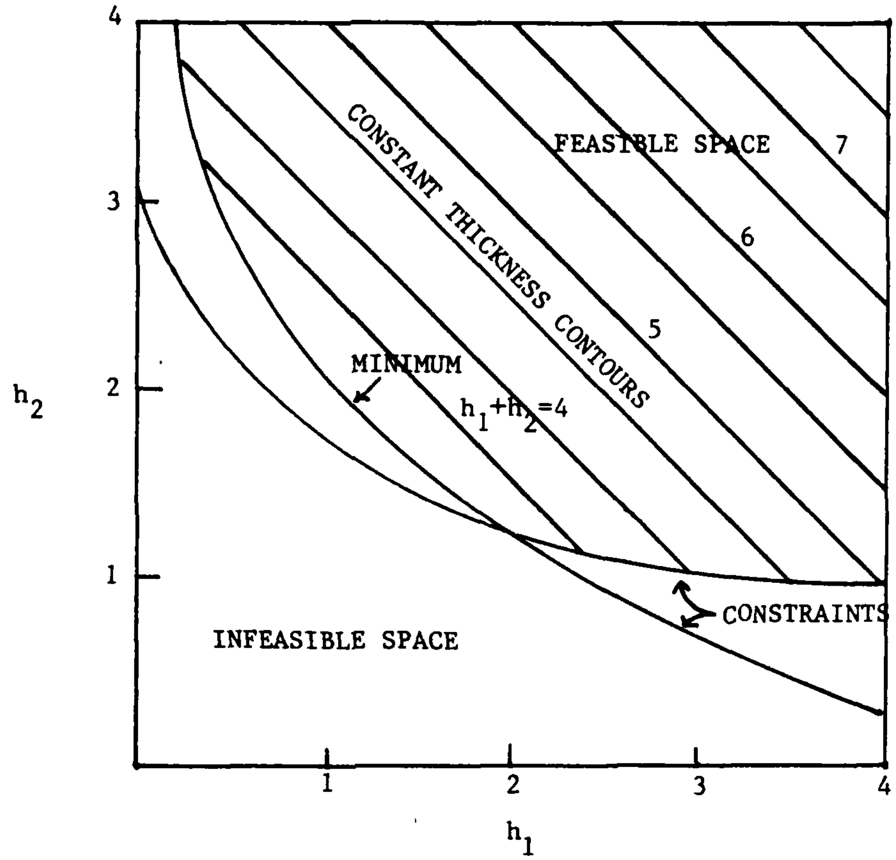


FIGURE 4: Definition of Optimization Terms

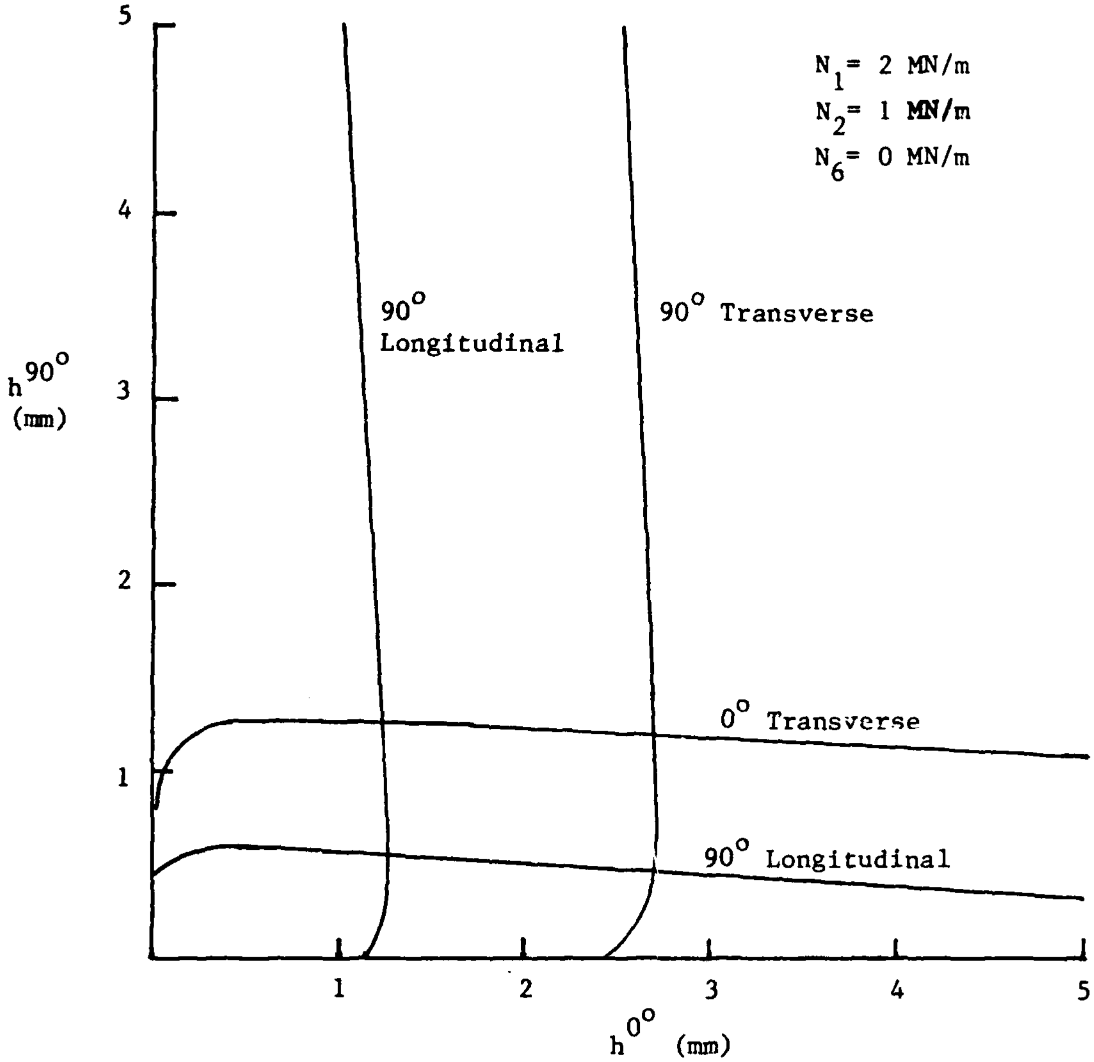


FIGURE 5: Failure Constraints for Maximum Strain Criteria

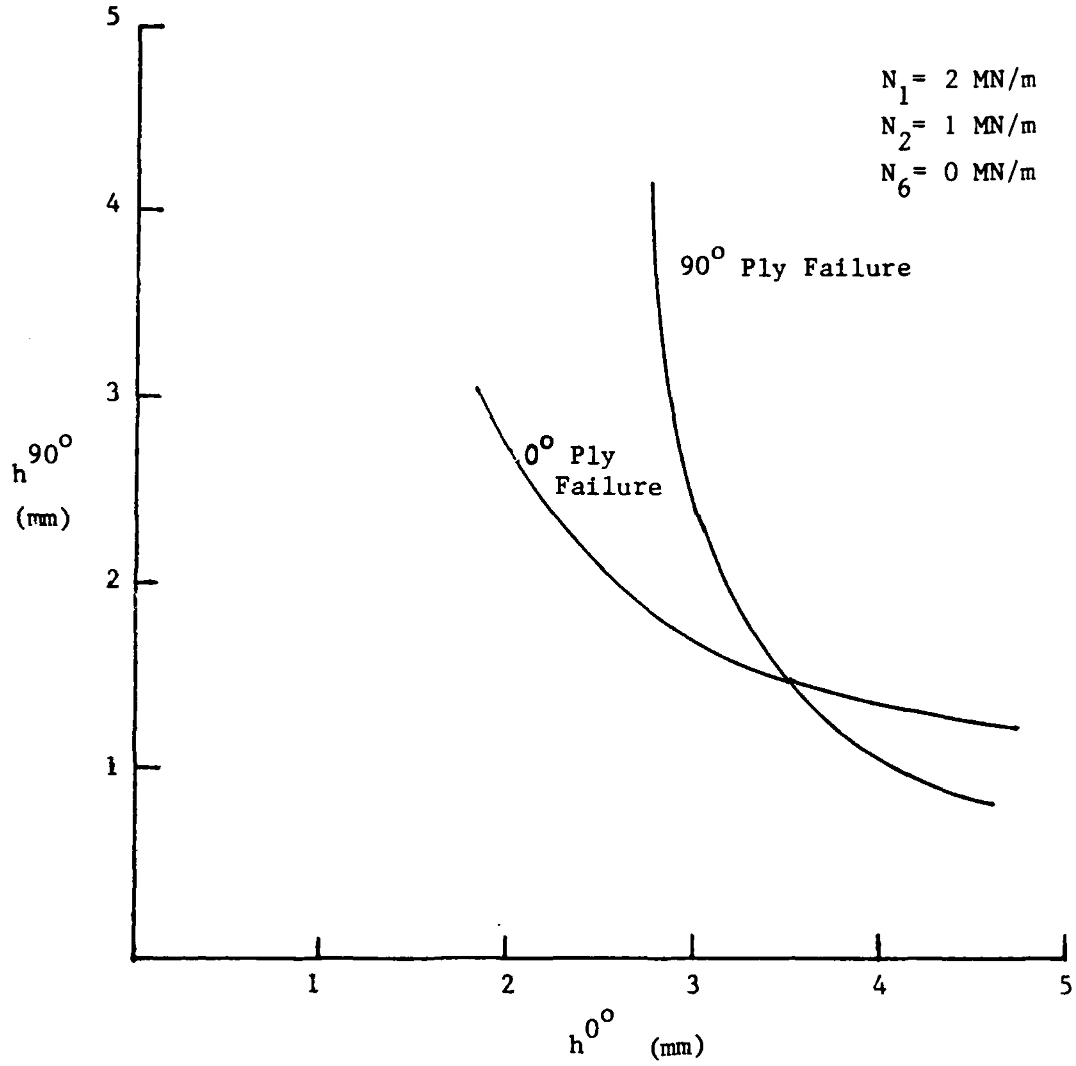


FIGURE 6: Failure Constraints for Quadratic Criteria

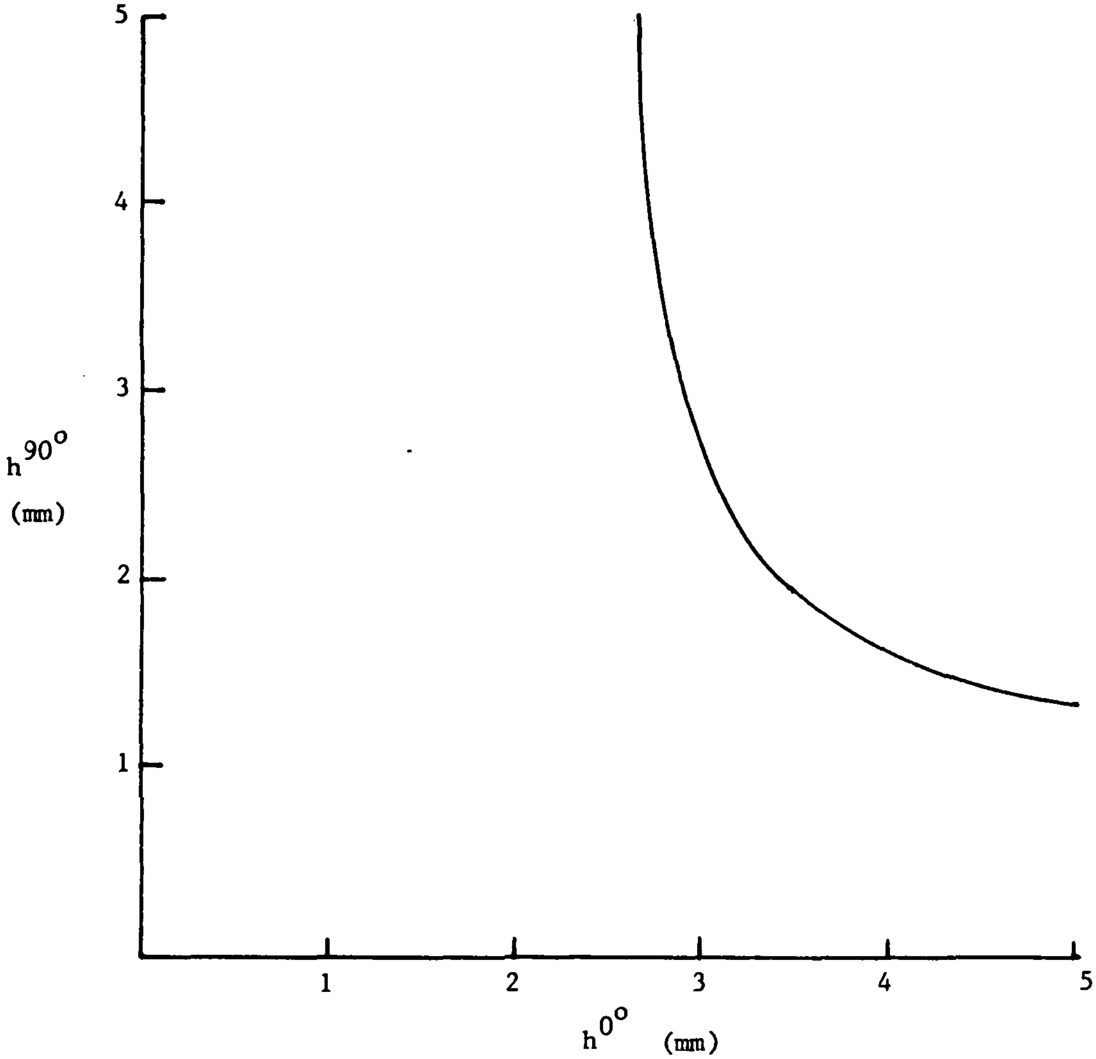


FIGURE 7: Failure Constraints for Strain Sphere Criteria

II. OPTIMIZATION METHODS

Ply Ratios

The laminate sizing problem can be stated in the language of optimization theory as follows;

find min. of h

where

$$h = \sum_{i=1}^{m} h_i \qquad (12)$$

subject to

$$C_{P,L} \leq 0 \qquad P,i = 1,2....m$$

$$h_i \geq 0 \qquad L = 1,2.....n_\ell$$

where

$$C_{P,L} = \vec{\varepsilon}^{\,T}_{(L)} \, |G^{(P)}| \, \vec{\varepsilon}_{(L)} + \vec{G}^{(P)^T} \varepsilon_{(L)} - 1$$

$|G^{(P)}|$ -quadratic failure criteria parameters transformed from the orientation of ply group P

$\vec{G}^{(P)}$ -linear terms of failure criteria transformed from the orientation of ply group P

$\vec{\varepsilon}_{(L)}$ -component of strain due to loading L

Although simply stated, there is no simple solution. One of several non-linear optimization methods could be applied to the problem. A modification of the method of feasible directions was chosen after examining ways to speed the computations enough so that solution on a microcomputer could be practical. The modification of the method makes

use of certain closed form equations at intermediate steps, reducing the number of calculations needed. The algorithm also takes advantage of the linearity of the objective function in terms of the design variables. This simplification also speeds up the algorithm as compared to more general formulations.

Although many figures in this section show the optimization process on two-dimensional graphs in design space, it's important to realize that some aspects of the problem may not be evident until 3 or more dimensions are considered. For example, the constraints may form long, narrow valleys that the search method must follow efficiently. Because all mathematics are derived in vector form, the extension to higher dimensions is simply a matter of book keeping for the computer.

Design optimization must always take into account the issue of local versus global minima. From optimization theory, if the feasible space can be shown to be convex, then there is only a global minima [9]. An informal definition of convexity is that any two points in the space can be connected by a straight line which does not pass out of the space at any point. The intersection of convex spaces forms a convex space [9]. Thus, if each constraint surface is convex, then there is only one minima. From observation of actual plots for cases with 2 ply groups, the failure constraints of composites meet this requirement. No proof of the generality of this observation is offered, but the assumption that the optimization leads to a global minima from any starting point will be accepted in this thesis.

Due to the periodicity of trigonometric functions, there will not be a single minima when angles are varied. This is a severe handicap to making angles a design variable.

In the method of feasible directions, the design is changed so that the trajectory in design space follows the constraint surfaces along a direction that decreases the objective function as quickly as possible, but never leaves the feasible region. A non-linear constraint cannot be followed continuously because, numerically, the algorithm must take finite, linear steps. Therefore, a vector is found which both decreases the objective function and does not violate the constraint for a finite move. The trajectory of a feasible direction algorithm is shown in Figure 8.

The problem with this method, for our purposes, is that finding the distance to the next constraint along an arbitray vector requires a numerical, one-dimensional search. Since each constraint evaluation requires forming the laminate A matrix, inverting the matrix, solving for strains, and evaluating the failure equations, we would like to reduce the number of iterations required for this search. Some approximations were tried, based on assuming the inverse of strain to be a linear function of ply group thickness. These were meant to speed the search, but were not found to be completely reliable. Instead, the method was modified to allow for larger error bands in the numerical search.

Briefly, the modification consists of measuring the distance across the constraint surface "valley", along a vector on which the objective function is a constant. This restricts the method to problems with an objective function that is linear in terms of the design variables. Finding this distance still requires a numerical one-dimensional search, such as bisection, but now the error band can be quite large, reducing the number of iterations needed. The larger error band is allowable because only a rough measure of the distance across is needed, whereas in the feasible directions method, the constraint surface must be

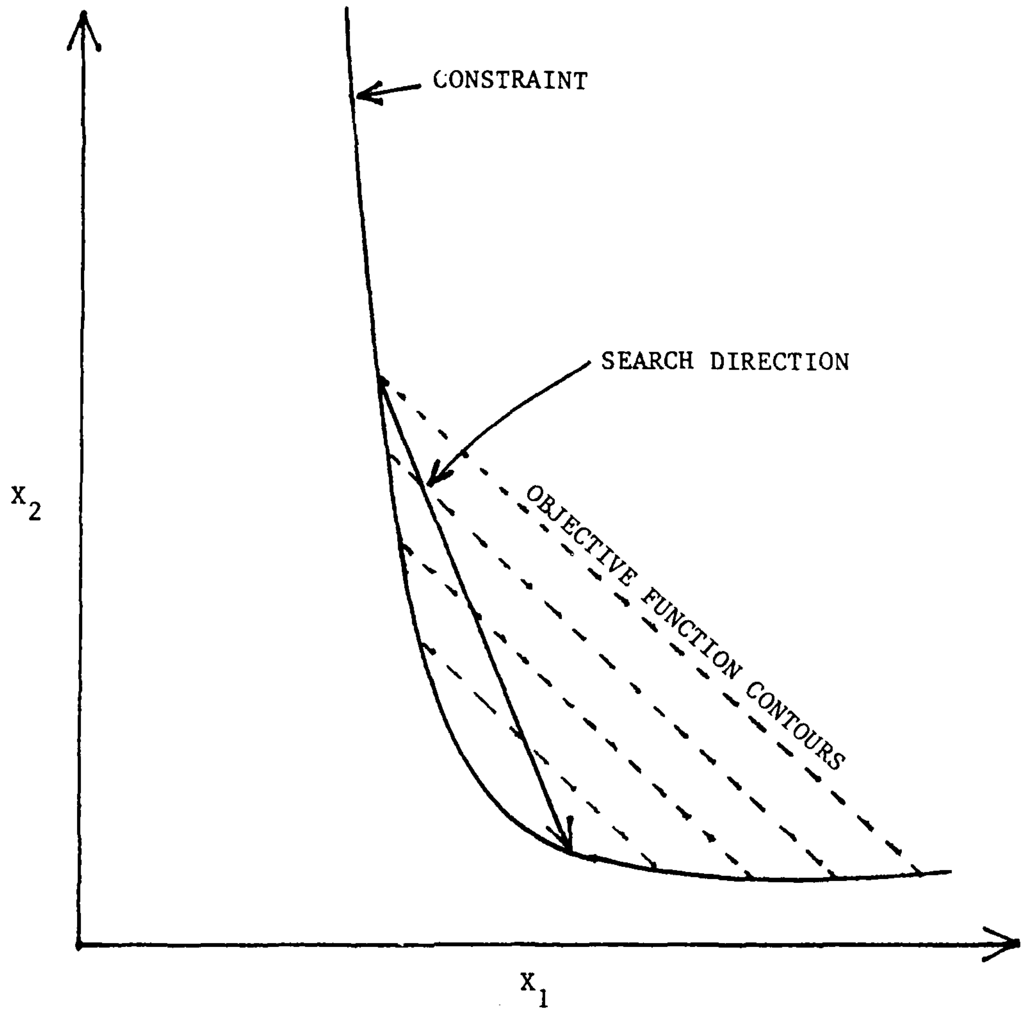


FIGURE 8: Trajectory for Method of Feasible Directions

located with high accuracy, since that point serves as the starting coordinate of the next iteration of the search. We assume the bottom of the "valley" will be about halfway across. From the halfway point, ply ratios are keep constant, and the total thickness of the laminate is scaled so that the coordinates in design space rest directly on the constraint surface defining the feasible region. The scaling operation is based on recognizing that for constant ply ratios, strain is proportional to total thickness. This closed form equation compensates for the error band of the numercial search. From the new coordinate, the procedure repeats until changes are very small, or a new search direction cannot be found (Kuhn-Tucker conditions for optimality [9]). A possible trajectory for the modified method is shown in Figure 9.

The constraint that thickness be greater-than or equal-to zero is known as a "side constraint". These linear constraints are simple enough to be handled by seperate logic. If the one-dimensional search hits a side constraint, and no strength constraints are violated at that point, the procedure stops on the h =0 plane, rescales the laminate, and proceeds as before. Any constraints associated with a zero thickness ply are ignored. Once a ply is set to zero thickness, it is never restored. The ability to completely drop a ply group's constraints seems to be unique to the programs developed for this thesis.

A step-by-step description of the algorithm will be presented, along with the relevant equations. For clarity, the variables used in this section will not always be identical to those actually used in the programs.

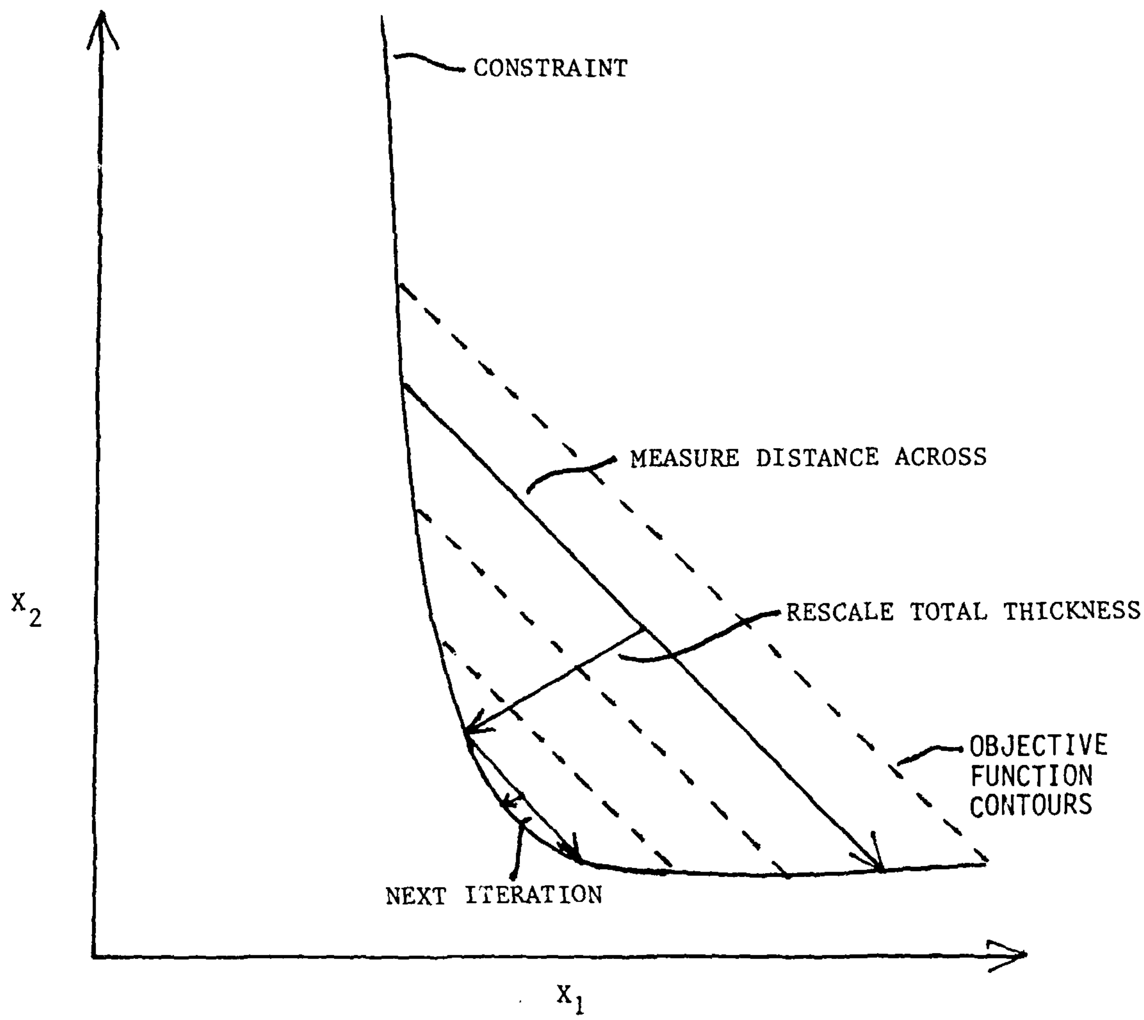


FIGURE 9: Trajectory for Modified Method

1) Laminate Scaling

Before any optimization of ply ratios can be considered, we must first be able to size the total thickness of a laminate with constant ply ratios. Strains are proportional to total thickness. This is evident by writting the stress- strain relation as

$$\vec{\varepsilon} = \frac{1}{h}[a^*]\vec{N} \tag{13}$$

where $[a^*]$ is the thickness normalized inverse of the A matrix. Instead of total thickness, it is more convenient to use the change in the distance from the origin in design space as the scaling parameter. The strain proportionality is the same for either parameter since

$$\frac{h}{h^o} = \frac{\Sigma\Delta h^o}{\Sigma h^o} = \Delta \; : \; \frac{r}{r^o} = \frac{\Sigma(\Delta h^o)^2}{\Sigma h^{o2}} = \Delta \tag{14}$$

where Δ is a proportional change of the individual ply thicknesses, and r is the distance from the origin. To use this linear relation, a reference strain vector is calculated, along with a reference r^o. Then, as long as ply ratios are constant, strain for any other value of r can be found from the equation

$$\varepsilon = \frac{r^o \varepsilon^o}{r} \tag{15}$$

where the superscript o refers to reference conditions. This relation can be substituted into any of the strain-space failure criteria. With the quadratic criterion we have

$$\frac{r^o}{r}^2 \vec{\varepsilon}^{oT}_{(L)} \left|G^{(P)}\right| \vec{\varepsilon}^o_{(L)} + \frac{r^o}{r} \vec{G}^{(P)^T} \vec{\varepsilon}^o_{(L)} = 1 - e_1 \tag{16}$$

where e_1 is a small (10^{-6}) offset that ensures the point stays slightly in the feasible region despite any numerical error. Solving for r

$$r = \frac{-B + B^2 - 4AC}{2A} \tag{17}$$

where

$$A = 1 - e_1$$

$$B = -\vec{G}^{(P)^T} \vec{\varepsilon}^{\,o}_{(L)} r^o$$

$$C = -\vec{\varepsilon}^{\,oT}_{(L)} \left|G^{(P)}\right| \vec{\varepsilon}^{\,o}_{(L)} r^{o2}$$

The value of r should be calculated for every possible constraint. The largest resulting value corresponds to the constraint forming the boundary between feasible and infeasible space. With this value of r, the ply group thicknesses are scaled according to

$$\vec{h} = \vec{h}^o \frac{r}{r^o} \tag{18}$$

where again, the superscript o means a reference condition.

2) Initial Feasible Point

Thicknesses are first set to a large, arbitrary value, to be assured of starting in the feasible region. The program sets all ply group thicknesses to $1/\sqrt{m}$ where m is the number of ply groups. Next, the total thickness of the laminate is scaled so that one constraint is critical (Figure 10). The scaling operation is given above.

3) Active Constraint List

At any step in the optimization, one or more constraints will be active. These are the constraints that are currently near critical as

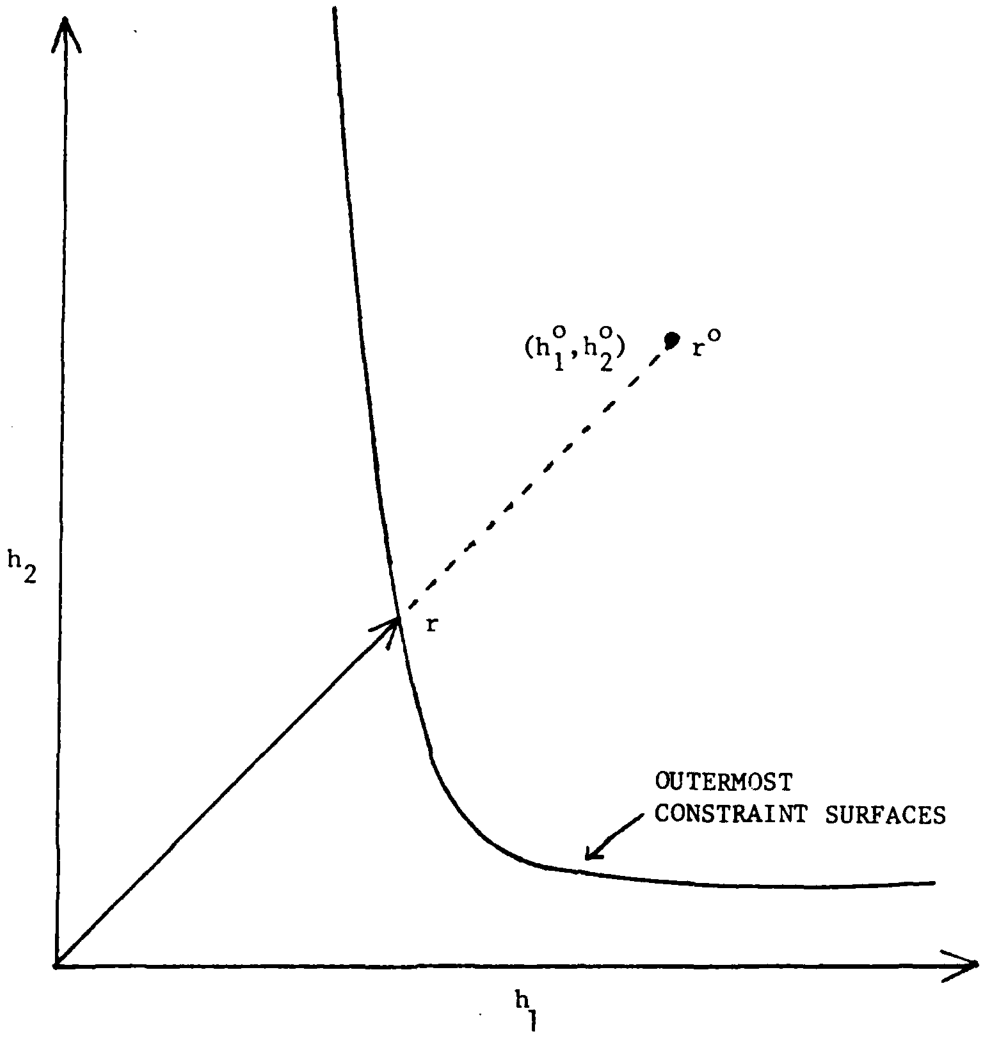


FIGURE 10: Scale Total Laminate Thickness

Ply ratios constant

defined by

$$\vec{\varepsilon}^{T}_{(L)} \; |G^{(P)}| \; \vec{\varepsilon}_{(L)} + \vec{G}^{(P)^T} \vec{\varepsilon}_{(L)} - 1 > e_2 \tag{19}$$

where a value of 0.05 has been found to work well for e_2. Before finding a search direction, the program must evaluate this equation for all values of P and L, and maintain a list of these values for which the constraint is active.

4) New Direction

We need to find a vector which points away from all the active constraints and is parellel to the constant total thickness plane (Figure 11). Components of the gradient vector are first calculated for each active constraint according to the equation

$$\frac{\partial C_{P,L}}{\partial h_i} = \vec{\varepsilon}^{T}_{(L)} \; |G^{(P)}| \vec{\varepsilon}_{(L),h_i} + G^{(P)^T} \vec{\varepsilon}_{(L),h_i} \tag{20}$$

where

$$\vec{\varepsilon}_{(L),h_i} = \begin{matrix} \partial\varepsilon_1/\partial h_i \\ \partial\varepsilon_2/\partial h_i \\ \partial\varepsilon_6/\partial h_i \end{matrix}$$

Since the applied loads are independent of the laminate configuration, the partials of strain can be evaluated from the stress-strain relation as follows;

$$0 = \frac{\partial}{\partial h_i} (|A|\vec{\varepsilon})$$

$$= |A|,_{h_i}\vec{\varepsilon} + |A|\vec{\varepsilon},_{h_i}$$

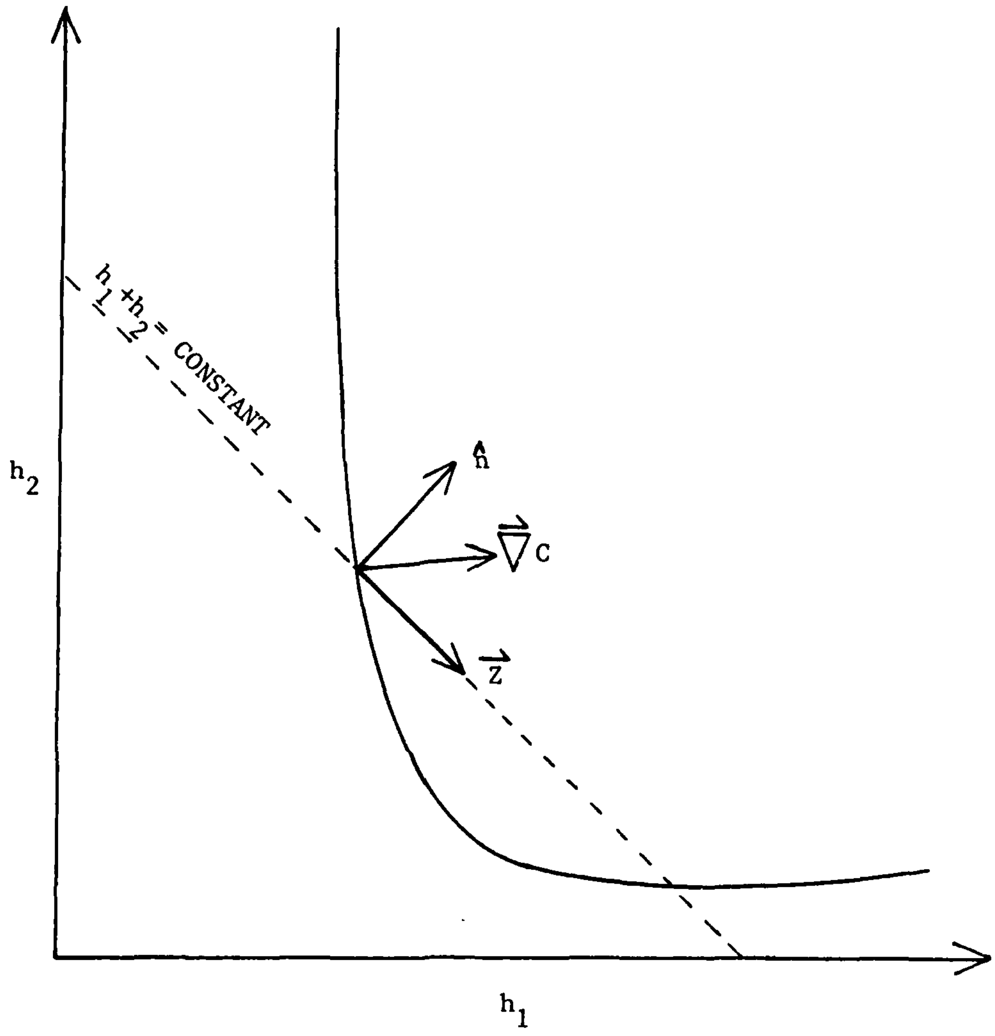


FIGURE 11: Search Direction

where

$$|A|,h_i = |Q^{(i)}|$$

so that

$$\vec{\varepsilon},h_i = -|A^{-1}|\ |Q^{(i)}|\vec{\varepsilon} \tag{21}$$

The gradient of each active constraint is normalized to unit length. The individual gradients are then summed and the result is normalized to a unit length. The reason for summing the gradients can only be visualized in 3 dimensions. Suppose two constraint surfaces meet to form a valley, and the objective function can still be reduced by following the valley along its length. If only one constraint were operated on at a time, the trajectory would bounce inefficiently back and forth between the surfaces. By summing the normalized vectors, a resulting vector that points down the valley can be formed. The negative of the summation will point into feasible space. This resultant vector will be called $\vec{W}$.

The projection onto the constant thickness plane is done by the double cross-product

$$\vec{z} = \hat{n} \times (\vec{w} \times \hat{n})$$

which, by a vector indenity can be written

$$\vec{z} = \vec{w} - (\vec{w} \cdot \hat{n})\hat{n} \tag{22}$$

where $\hat{n}$ is the unit normal to the plane defined by

$$\sum_{i=1}^{m} h_i = \text{constant} \tag{23}$$

In keeping with good numerical practice, $\vec{Z}$ is also normalized. If the length of $\vec{Z}$ before normalization is small (10^{-6}) then $\vec{W}$ and $\hat{n}$ must be near parallel. This would indicate that a minimum has been reached and

the program halts.

5) Distance to Next Constraint

The next step is to find the distance along $\vec{Z}$ to the next constraint (Figure 12). A bisection method is used for the one-dimensional search. The vector $\vec{Z}$ describes relative changes in the ply group thicknesses. Moving a scalar distance S along $\vec{Z}$ changes the thicknesses according to

$$\vec{h} = \vec{h}^{o} + s\vec{Z} \tag{24}$$

where $\vec{h}^{O}$ is the vector of current thicknesses for S=0. Note that even though the individual ply groups are changing, total thickness stays constant along $\vec{Z}$. The program will need to be able to quickly calcuate the A matrix as ply groups change. To save a few multiplications, the programs represents A as

$$|A| = |A_o| + s|A_z| \tag{25}$$

where

$$A_{oij} = \sum_{k=1}^{m} Q_{ij}^{(k)} h_k^o$$

$$Az_{ij} = \sum_{k=1}^{m} Q_{ij}^{(k)} Z_k$$

The initial bounds on the bisection search are S=0 and S=Smax where Smax is the distance to the nearest h= 0 constraint. Smax is calculated by finding the largest positive value of the equation

$$S_{max} = -h_i/Z_i \qquad i = 1,2,....m \tag{26}$$

The usual bisection method is slightly modified. First, instead of trying to find the zero of a single equation, we must evaluate each

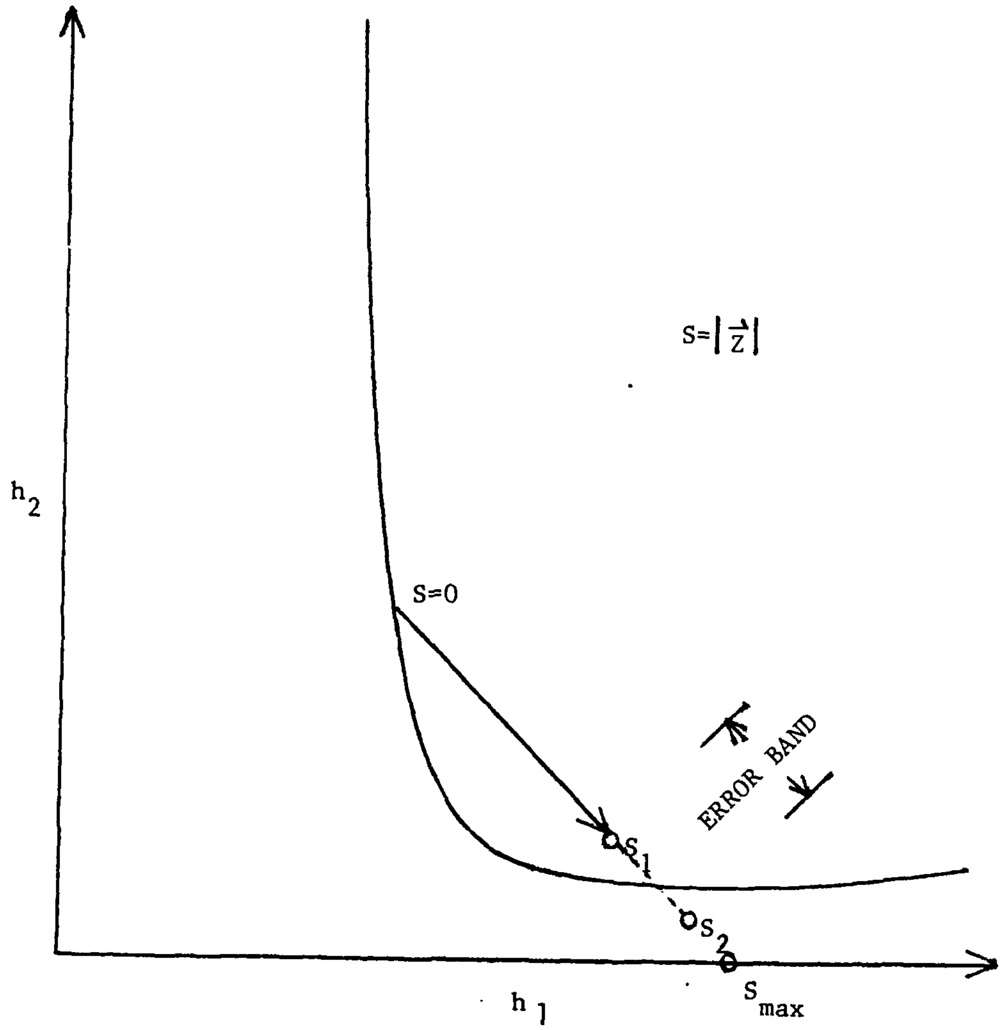


FIGURE 12: Distance to Next Constraint

possible constraint to find the boundary between feasible and infeasible spaces. The programs in the appendices contain a subroutine which evaluates the constraints and returns a single flag with the value "FAIL" if a single constraint is violated, and "PASS" if no constraint is violated ($C_{P,L} < 0$ for all P,L) The second feature is that S=Smax may be in the feasible region. What this means is that a ply group can be reduced to zero thickness without violating any constraints. If this is the case, the program updates the ply group thickness vector for the point S=Smax and rescales the laminate, eliminating constraints associated with the zero thickness ply group. The algorithm then restarts from step 2. If S=Smax is not feasible, then the bisection continues with the follow steps:

1) Let S1=0, S2=Smax
2) Let S=(S1+S2)/2
3) Test all constraints at point S
4) If flag="PASS" then S1=S
   If flag="FAIL" then S2=S
5) If $S2-S1<10^{-5}$ then search direction immediately hits constraint. This indicates the minimum has been found.
6) If (S2-S1)/S1>1/4 then go to step 2. Else stop bisection procedure

Step 6 checks to see if the error with which the distance to the constraint is known, is less than 1/4 the distance across the "valley". The 1/4 is arbitray, but gives good overall convergence of the algorithm with a minimum number of bisection iterations. Note that for each value of S tested, the A matrix must be formed, inverted, strains calculated, and constraint evaluated.

5) Rescale Laminate

Once the distance to the next constraint is known, we take S=S1/2. From this point in design space, the total thickness is reduced by the laminate scaling procedure (Figure 13). If the change in total thickness is small (less than 1/10 a single ply thickness), the algorithm is assumed to have reached a minimum and halts. If not, the algorithm repeats from step 2. The loop continues until one of the halt conditions is reached.

The organization of the program is shown by a flowchart in Figure 14. The flowchart is only meant to be an aid to understanding the steps required. The interconnections between subroutines in the actual programs are somewhat more complex.

Table 1 gives some examples of the convergence rate and number of inverse A matrix evaluations (the most time consuming step) required for the optimization. Times are given for a ZX-81 computer which has a Z-80 microprocessor. An iteration is counted as the total loop from step 2 to 5. Three or 4 iterations is typical unless some ply groups are going to zero thickness, which counts as a full iteration.

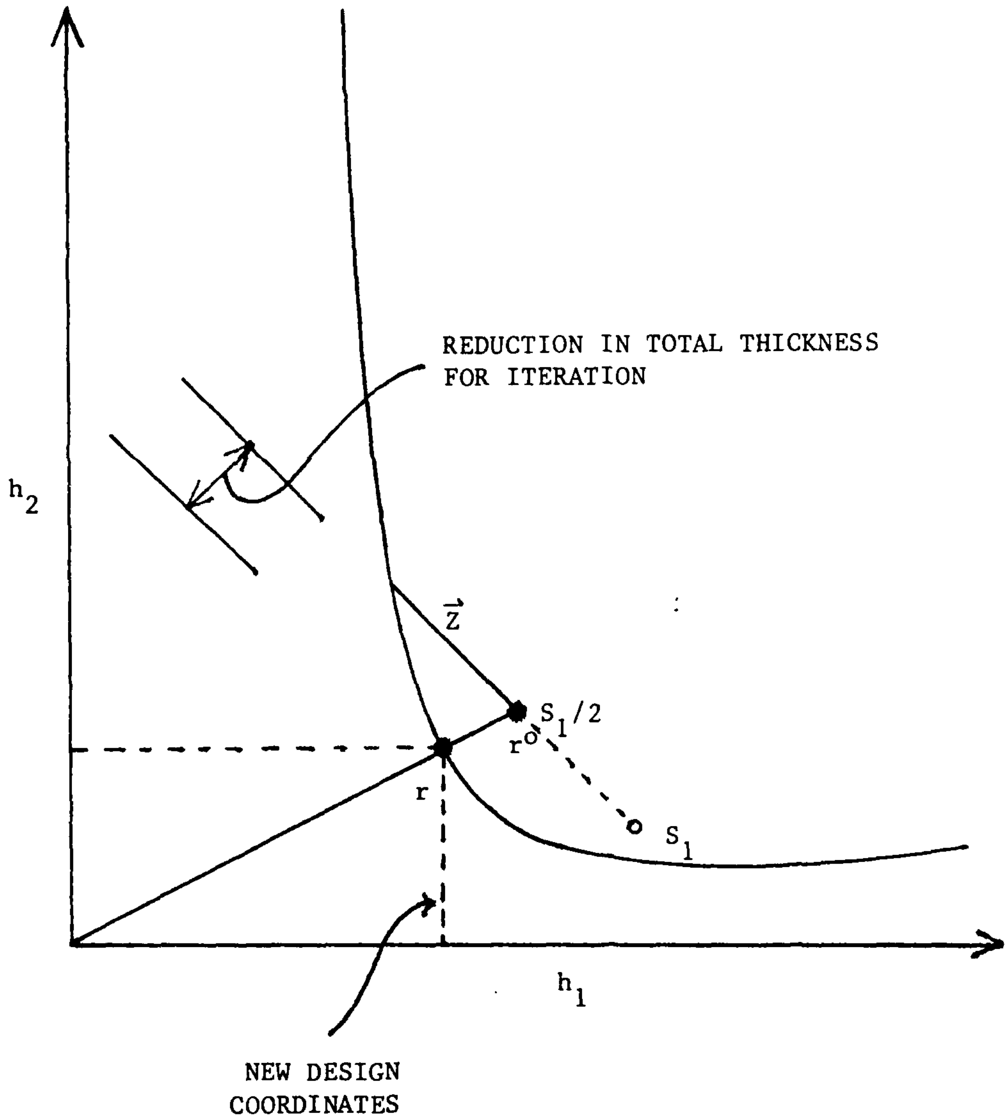


FIGURE 13: Rescale Total Thickness

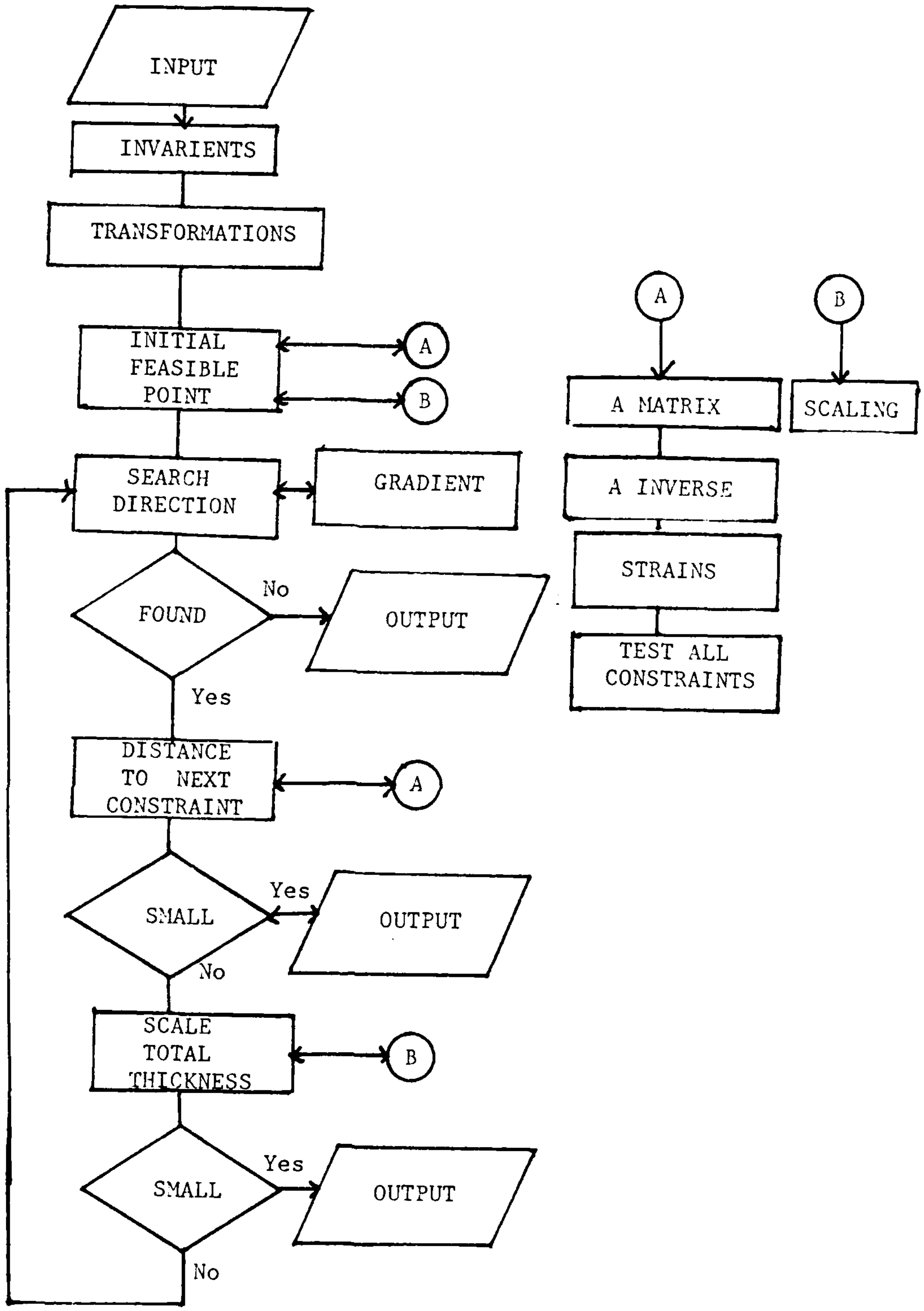


FIGURE 14 : Flowchart

Laminate	Load Vectors MN/m	Active Constraints	Iterations	Matrix Inversions	Sec
0/90	2,1,0	2	3	27	164
0/90	2,1,0 1.75,0.5,0.3	3	6	58	250
0/90/+45	2,1,0	4	3	26	264
0/90/ +30/+60	2,1,0	6	4	34	555
0/90/ +30/+60	2,1,0 1.75,0.5,1 -3,1,0	6	5	40	930
0/90/+15 +30/+45/ +60/+75	5,1,0	7	2	19	480

TABLE 1: Algorithm Performance

Continuously Variable Angles

The most obvious approach to selecting appropriate ply orientations is to let the computer calculate the optimal values. There is no fundamental reason why this cannot be done, but there are some implementation problems, and the results are not always satisfactory. There are several mathematical difficulties in optimizing for best ply orientations. First, the objective function (total thickness) is not directly a function of angle. Second, there may be many local minima. Third, if a direction vector is found in the combined angle and thickness space, the magnitude of the scalar distance will have different meaning for each type of design variable. Finally, there is the

the practical difficulty that ply orientations cannot be completely arbitrary due to manufacturing limitations. There should be some minimum angular step size limited by the lay-up procedures used. The algorithm derived here, while not completly satisfactory, attempts to account for all these difficulties.

The approach taken is to first divide the problem into a multi-level optimization [10], where angles and ply ratios are optimized independently. We can alternate between the two types of optimization until the laminate converges to a minimum thickness design. Ply thicknesses are handled exactly as before, with the given angles held constant. During the angle optimization, ply ratios are held constant and the angles varied to minimize total thickness.

The angle optimization used here is not a direct method like that used for the thicknesses, but instead relies on minimizing a related, unconstrained function with the assumption that total thickness will

decrease at the same time. One approach is to chose a function which will lead to the simultaneous failure condition, which should result in an efficient laminate. Another desirable feature is that the results should not be too sensitive to the selection of initial angles. After experimenting with several possible functions, the best was found to be the variance of the all the constraints, given by the equation

$$\sigma = \frac{1}{n_c} \sum_{P=1}^{m} \sum_{L=1}^{n_e} C_{P,L}^{\;2} - \frac{1}{n_c^{\,2}} \sum_{P=1}^{m} \sum_{L=1}^{n_e} C_{P,L}^{\;2} \tag{27}$$

where $n_c = m \cdot n_\ell$

If this function where minimized to a value of zero, a simultaneous failure condition for the laminate would be reached. In cases with multiple loads, simultaneous failure for all loads is usually impossible, but we assume that as the variance is minimized, as many constraints will become active as possible. It will be shown that simultaneous failure is not always the optimal condition for a composite laminate, but for most cases it will either be the minimum or at least very close to the minimum thickness. Before trying the variance, the author had attempted to minimize the value of the largest current constraint function. This necessitated finding some way to handle multiple constraints that had nearly the same value. This version of the program often terminated early because a satisfactory way was never derived for finding a common vector that would reduce the value of more than one constraint simultaneously. To find the minimum of the variance, a steepest descent method was used. Normally, steepest descent is considered the least efficient way to minimize an

unconstrained function, but it was found to be sufficient for the current research. The program should be modified in the future to include a conjugate gradient method [10].

The steepest descent, along with most other search methods, needs the value of the gradient. Terms of the gradient are given by

$$\frac{\partial\sigma}{\partial\theta_i} = \frac{z}{n_c} \sum_{P=1}^{m} \sum_{L=1}^{n_\ell} C_{P,L} \frac{\partial C_{P,L}}{\partial\theta_i} - \frac{1}{n_c} \sum_{P=1}^{m} \sum_{L=1}^{n_\ell} C_{P,L} \Sigma \Sigma \frac{\partial C_{P,L}}{\partial\theta_i} \tag{28}$$

where

$$\frac{\partial C_{P,L}}{\partial\theta_i} = Z\vec{\varepsilon}^{\,T}_{(L)} \; |G^{(P)}|\vec{\varepsilon}_{(L),\theta_i} + \vec{\varepsilon}^{\,T}_{(L)}|G^{(P)}|_{,\theta_i}\vec{\varepsilon}_{(L)}$$
$$+ \vec{G}^{(P)^T}\vec{\varepsilon}_{(L),\theta_i} + \vec{G}^{(P)^T}_{,\theta_i}\vec{\varepsilon}_{(L)} \tag{29}$$

and

$$\vec{\varepsilon}_{(L),\theta_i} = -|A^{-1}| \; |A|_{,\theta_i}\vec{\varepsilon}_{(L)}$$
$$|A|_{,\theta_i} = |Q^{(i)}|_{,\theta_i} h_i$$

It should be noted that

$$|G^{(P)}|_{,\theta_i} = 0 \text{ for } i \neq P$$

The angular derivatives of the Q's and G's are given in Appendix A. The negative of the gradient will form the search direction. The scalar distance along the search direction is found by taking discrete steps and stopping when the thickness begins to increase (and then taking one step backwards). Because the variance is only a function related to the

actual minimum, we do not determine the distance by the magnitude of the variance, but instead, the function we are actually interested in. Thickness is calculated by using the scaling equations developed previously. More efficient one-dimensional search methods will have difficluties with the multiple local minima.

The steps are taken so that all the angles change by some minimum step. To maintain the minimum step size, the angles are incremented by the equation

$$\theta_i^{k+1} = \theta_i^k + \{\mathrm{CINT}\ [(k+1)Z_i] - \mathrm{CINT}\ (kZ_i)\}\Delta\theta$$

where CINT implies taking the closest integer value and k is an incremental step counter. The direction vector $\vec{Z}$ is normalized by its largest element. At each unit increment of k, the angle corresponding to the largest element of $\vec{Z}$ is incremented by $\Delta\theta$. Other angles may not be incremented at each step, depending on the relative values of the $\vec{Z}$ vector elements. Thus, the direction vector is not followed exactly, but rather on a broken path. The amount of divergence from the search direction is determined by the value of $\Delta\theta$. If the angle start out as multiples of 10^{o} and $\Delta\theta$ is 10^{o}, then the angles will stay as multiples of 10^{o} throughout the search.

The overall procedure for the multi-level optimization can be summerized as follows:

1) Enter loads and starting angles
2) Find a search direction based on the variance
3) Perform a one-dimensional search to minimize total thickness with constant ply

ratios

4) Repeat from step 2 until no further changes in angle can be made

5) Optimize the ply ratios

6) Repeat from step 2 until neither type of optimization can make further progress

Testing of the program shows that one pass through steps 1 to 6 is all that is needed. Usually, the angle optimization brings enough of the constraints to critical values that the ply ratio optimization can make little progress. In turn, after the ply thickness routine is finished, there is little the angle optimizer can change.

Typically, the angle optimization will need 4-6 search directions to converge, requiring 10-20 minutes for 4 ply groups and a pair of independent loads.

Orthotropic Laminate

A designer may not want a general symmetric laminate. He may be more comfortable with an orthotropic laminate which eliminates the shear coupling terms and allows the use of many existing orthotropic plate analysis equations. An orthotropic laminate can be made by keeping the laminate balanced. That is, for every ply at $+\theta$, there is one at $-\theta$. There may also be manufacturing reasons for wanting a balanced laminate, such as filament winding operations. There is no difficulty in constraining the optimization procedure to yield balanced laminates. Most sophisticated optimization programs allow design variables to be coupled so that they maintain the same value. A simpler approach is to enter only the positive angle and set the A_{13} and A_{23} terms to zero. The resulting thickness found for the positive angle must then be split between the positive and negative angles in the actual laminate. With the reduced A matrix, a faster matrix inversion can be written.

When designing with orthotropic laminates, the orthotropic axis should not be selected arbitrarily. For a single load, the orthotropic axes should be aligned with the principle axes of the load. With mulitple loads, the selection is not so obvious. Finding the best axes with respect to the load is a much simpler problem than the general optimal angle search discussed above. A search for the best axes can be reduced to a one-dimensional search. The procedure can be thought of as finding the best rigid body rotation of the laminate with respect to the loads while performing a thickness optimization of each rotation angle (Figure 15). For computational simplicity, the program actually rotates

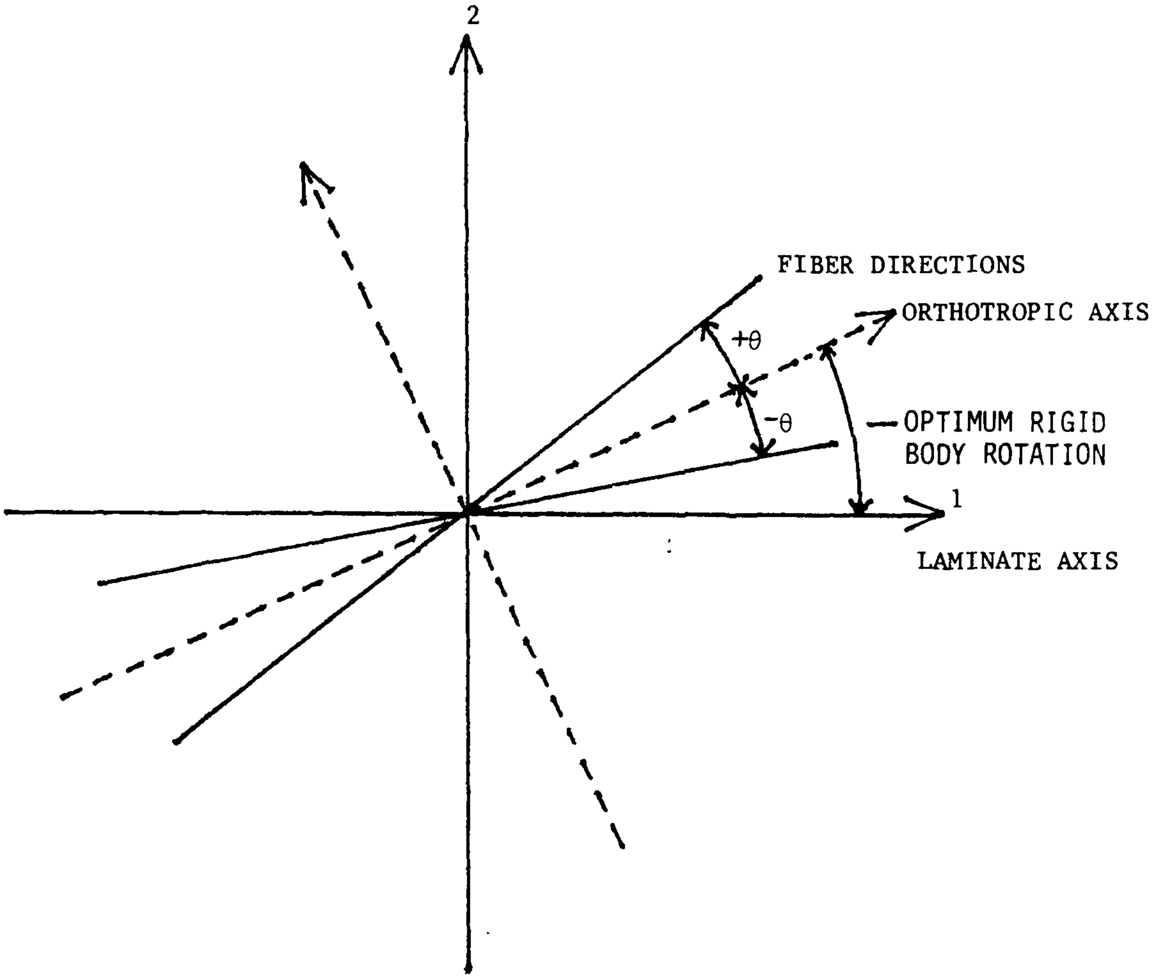


FIGURE 15: Definition of Angles for Orthotropic Laminate

the loads and keeps the laminate angles fixed. Even this one-dimensional search could be time consuming without a fast ply ratio optimization algorithm. The orthotropic optimization with the strain-sphere failure criteria is fast enough to make a search for best orientation practical.

The search proceaure can be summerized as follows:

1) Enter initial laminate angles, loads, bounds on search angle, and maximum error for search.

2) Divide the bounded region with 4 equally spaced points, with endpoints on the bounds

3) Find the minimum laminate thickness at each point by rotating the loads by the negative of the current test angle

4) Check for the smallest value of the 4 thicknesses. The 2 points on either side of the smallest one become the new bounds.

5) If the bounds are greater than the maximum error, repeat from step 2. Only 2 new points need to be calculated.

The method being used here is very similar to the bisection method for finding the zero of a function. Bisection requires 3 function values in order to reduce the size of the region the zero can be in. Here, a fourth point is needed because we are searching for the zero of the first derivative instead of a zero of the function.

III. APPLICATION

Examples

A few illustrative examples will be discussed to demonstrate the operation of the optimization procedures. A detailed comparison of the weight savings possible with ply ratio optimization, angle optimization, and no optimization will be given in the next section.

The strength ratios defined in [7] will be needed to show which plies are critical for given loads. The ratio is defined as the value of R in the equation

$$R^2 \vec{\varepsilon}^T |G| \vec{\varepsilon} + R G^T \vec{\varepsilon} = 1$$

An R of 1 means the ply is at the boundary of the failure envelope. R's 1 mean the ply is in the safe region on the failure envelope. The R's can be interpreted as the ratio of the applied load vector length to the maximum load vector length.

Most of the examples presented here will use T300/5208 as the material. Properties of this material along with Kevlar and aluminum (used in certain examples) are given in Table 2. Figure 16 is an example output from an Epson HX-20 microcomputer. Only ply ratios are being changed and the angles are given as 0/90/45/-45. This example demonstrates a case where there is no severest load condition. Looking at the strength ratios, we can see that the 90 and -45 plies are near failure for the first load condition,

Material Properties
T300/5208
EX= 181 GPa
EY= 10.3 GPa
ES= 7.17 GPa
VX= .28
X= 1500 MPa
X'= 1500 MPa
Y= 40 MPa
Y'= 246 MPa
S= 68 MPa
Ply Thickness .000125 m

LOADING 1
N 1= 3 MN/m
N 2= 1 MN/m
N 6= 0 MN/m
LOADING 2
N 1= 1.5 MN/m
N 2= 1.5 MN/m
N 6=-.5 MN/m

Total thickness=
.0735E-01 m.
58.76 Plies

ANGLE	RATIO	#PLIES
0	.4416	25.95
90	.1236	7.26
45	.1774	10.42
-45	.2574	15.12

STRENGTH RATIOS
1=ULTIMATE STRAIN
>1 IS SAFE
LOADING 1

PLY	
0	1.4078
90	1
45	1.2091
-45	1.0973

LOADING 2

PLY	
0	1.0355
90	1.4004
45	1.0331
-45	1.4071

LAMINATE STRAINS
LOADING 1
e1=+3.628E-03
e2=+1.274E-03
e6=+0.691E-03
LOADING 2
e1=+1.224E-03
e2=+3.334E-03
e6=-2.157E-03

Norm. |A| in GPa.

106.207	20.035	-3.429
20.035	51.673	-3.429
-3.429	-3.429	24.309

Compliance (normalized)
in 1/TPa.

10.178	-3.887	0.887
-3.887	21.020	2.417
0.887	2.417	41.604

ENGINEERING CONSTANTS

E1= 98.3 GPa
E2= 47.6 GPa
E6= 24.0 GPa

v21= 0.382
v61= 0.087
v16= 0.021

FIGURE 16: Printout for Example Problem

Material	Ex	Ey	Vx	Es	Fiber Vol.
	Elastic Modulus in GPa				
T300/5208 Graphite/Epoxy	181.0	10.3	0.28	7.17	0.70
Kevlar 49/Epoxy	76.0	5.5	0.34	2.30	0.60
Aluminum	69.6	69.6	0.34	26.5	---

	X	X'	Y	Y'	S
	Strength in MPa:				
T300/5208 Graphite/Epoxy	1500	1500	40	246	68
Kevlar 49/Epoxy	1400	235	12	53	34
Aluminum	400	400	400	400	230

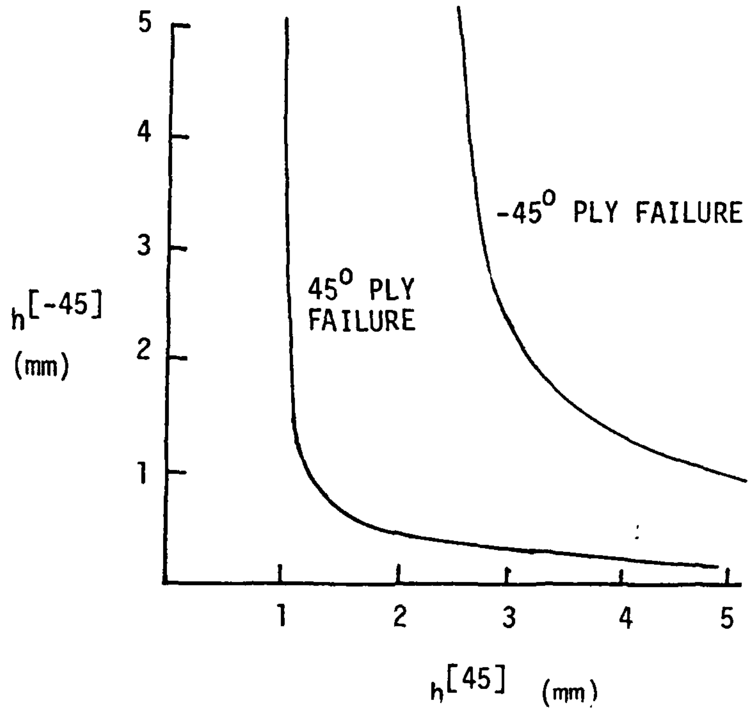
TABLE 2: Material Properties

and the 0 and +45 plies are near failure for the second load condition. The normalized A matrix shown in the output is defined as $|A|/h$ and the normalized compliance matrix is the inverse of $|A|$ times h.

The example given in Figure 17 is a case where simultaneous failure is impossible. The constraint curves in design space are plotted to show that one constraint is never on the boundary between the feasible and infeasible regions. The impossibility of simultaneous failure is also evident by examining the failure envelopes in strain space. The failure envelopes for graphite epoxy only intersect in the first and fourth quadrants (Figure 3). Pure shear transformed to principle strains is in the second or fourth quardrant. Even though one ply is never near failure, removing that ply increases the total thickness required.

Table 3 compares the results of optimization based on the strain-sphere approximation and the usual quadratic interaction criteria. The ply ratios are quite close, demonstrating that for loads in the first quadrant, the results are not sensitive to the particular criterion. Although the approximate criterion works well, all results presented elsewhere in the thesis will be based on the quadratic criteria unless otherwise stated. No detailed description of the algorithm for optimization with the strain sphere is given, but the method is almost identical to that used for the quadratic criteria. The major differences are that the gradient is redefined and the criterion only needs to be evaluated for the laminate as a whole, rather than for each ply individually.

Table 4 is an example of the orthotropic laminate optimization with optimal rigid body rotation. The best orthotropic axes could not have been selected from inspection of the load principle axes. The



LOAD
N1= 0
N2= 0
N6= 2 MN/m

Angle	Ply Ratio	# Plies Needed
45	.656	26.9
-45	.344	14.1

Strength Ratios

Angle	R
45	2.42
-45	1.00

Figure 17: Constraint Curves and Optimization Results for 45 Under Pure Shear

LOADS

N1= 4 MN/m	N1'= 2.76 MN/m
N2= 1 MN/m	N2'= 2.24 MN/m
N6= 0 MN/m	N6'=-1.48 MN/m

	# Plies Needed	
Ply Group	Tsai-Wu	Approximate
0	35.2	35.2
90	7.5	7.4
45	9.9	10.8
-45	33.8	33.7
Total	86.5	87.1

TABLE 3: Comparison of Approximate Strain-Sphere to Tsai-Wu Criteria for Optimization

Two independent Loads

LOADS

N1= 2 MN/m	N1'= 1.25 MN/m
N2= 1 MN/m	N2'= 1.75 MN/m
N6= 0 MN/m	N6'= -.43 MN/m

	# Plies Needed	
Angle	Fixed Axis	Variable Ortho. Axis
0	17.2	11.2
90	17.2	3.9
45	8.6	16.3
+45	8.6	16.3
Total	51.6	47.7

TABLE 4: Comparison of Optimal Orthotropic Laminates with Fixed and Variable Orthotropic Axis.

Optimal orthotropic axis at -30^{o} .

results for an orthotropic laminate with the axes arbitrarily set on one of the load principle axes are also given. The difference is substantial. Both examples are based on the maximum strain-sphere criterion.

Angle optimization is only needed if there is more than one independent load. For a single load, the algorithm will simply rotate the plies so that they lie on the load principle axes. This characteristic shows that there is more than one minima, since an angle-ply (consisting of a $+\theta$ and a $-\theta$ ply group) is more efficient than a cross-ply laminate (0's and 90's only). The program does not converge to the angle-ply solution unless the initial angles are close to the final value. We cannot predict the result when multiple loads are included. To show the relationship between load principle axis and optimized ply orientations, 2 independent loads that fall on the same Mohr's circle have been used as the design requirements. The loads and ply orientations can be superimposed on the same Mohr's circle. Figure 18 reflects some of the symmetries of the optimized laminate. An interesting example of how non-intuitive composites can be is shown in Figure 19. Two equal magnitute uniaxial loads are entered with one of the loads rotated by -40 from the laminate axis. Instead of placing the plies on the principle axes, the computer has picked slightly different angles, which give a thinner laminate than if the principle axes had been used. The starting angles were 0/90/45/-45, but the angles have converged so that only 2 ply groups remain.

Although there is now a method for finding good ply orientations, we still need to know how many initial angles should be used, and their initial values. One reason the search based on constraint variance was selected is because it seems to be less sensitive to choice of initial

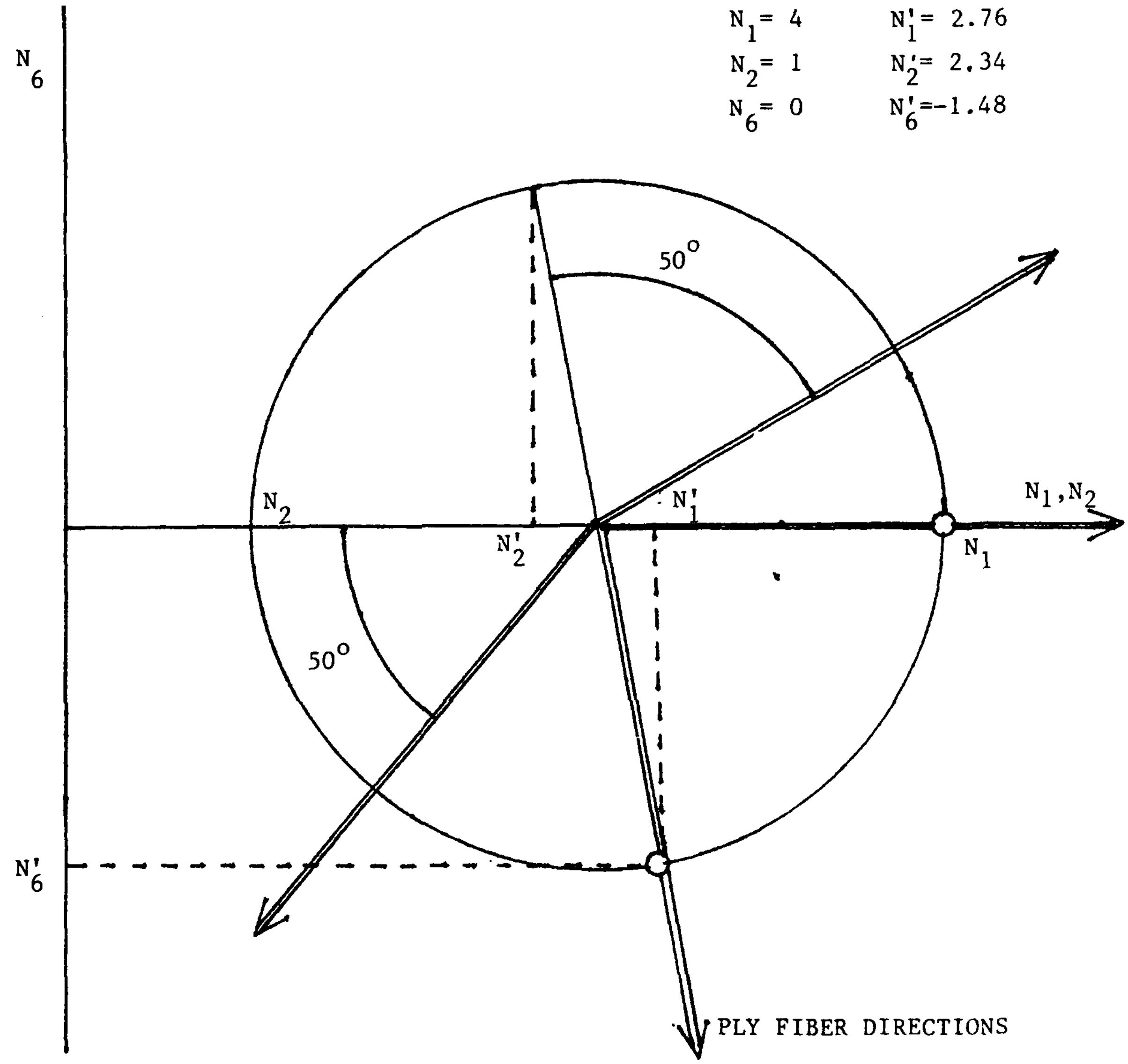


FIGURE 18: Mohr's Circle Representation of 2 Independent Loads with Superimposed Optimized Ply Orientations

Initial angles (0/90/±45)
All angles plotted as 2θ

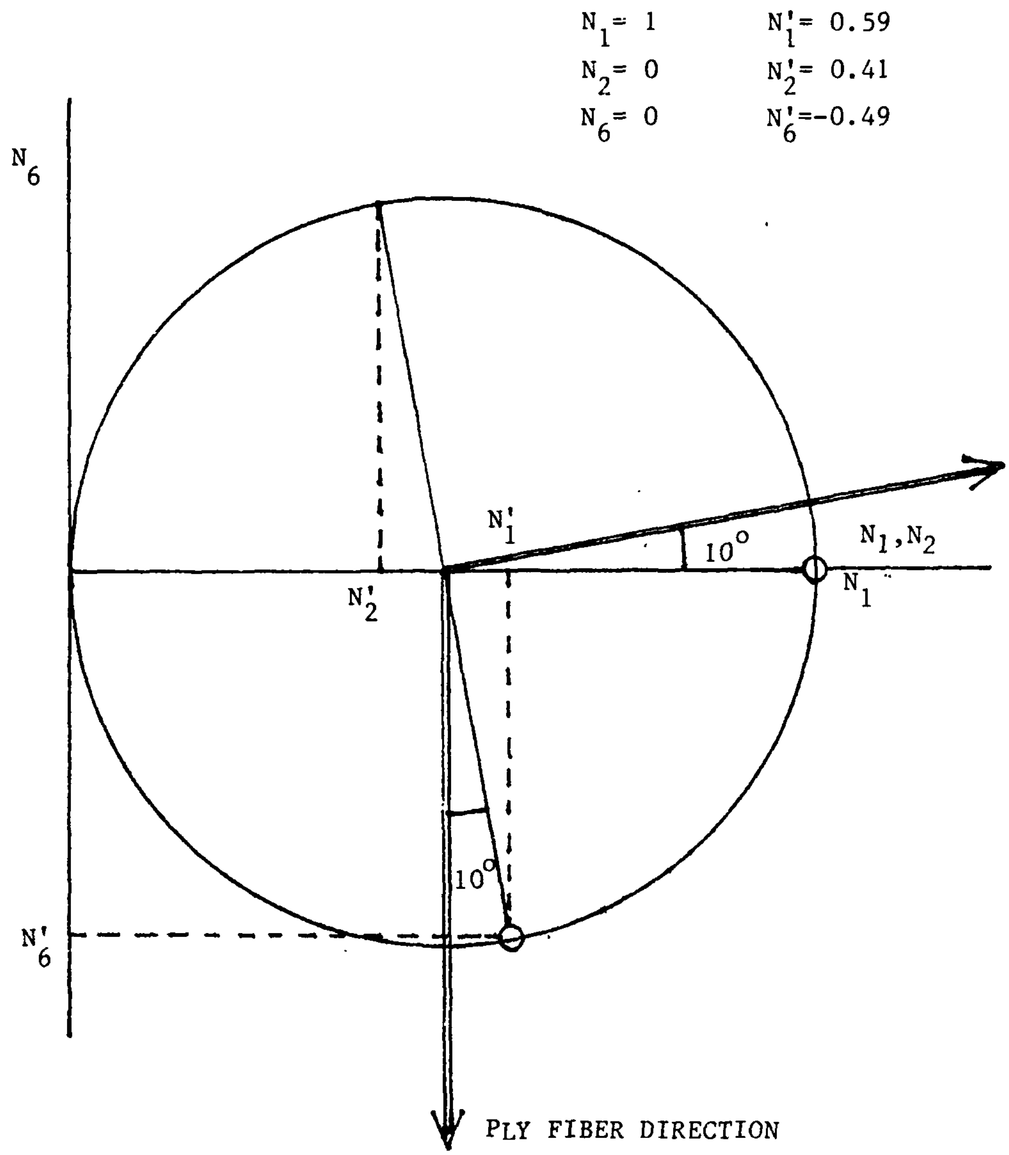


FIGURE 19: Mohr's Circle Representation of 2 Independent Loads with Superimposed Optimized Ply Orientations

Initial angles (0/90/±45)
All angles plotted as 2θ

angles than some of the other methods tried. The number of angles needed is still an open question. A quick look at gradient information suggests that too few angles (2 for example) will make the laminate sensitive to small changes in orientation or load. The 0/90/45/-45 starting point selected for all the above examples has been found to give effiecient laminates without the complexity of adding a lot of angles. Most of the examples run where with 2 loads, but a couple of cases were tried with 4 loads. The 4 ply group laminate was still adequate despite the additional loads.

All the examples given were run by applying the angle optimization first and then the ply ratio optimization. After the ply ratio optimization, no further attempt at changing the angles was made.

There is the possiblility that the combined angle/ply ratio optimization will yield a laminate with total thickness greater than would have been produced by ply ratio optimization alone. By bringing more constraints into play, the angle optimization may prevent the ply ratio program from making as much progress as it would have starting from some arbitrary initial angles. Often, the ply ratio program will not be able to change the laminate at all, leaving the ply ratios equal. From the evaluation presented later in this thesis, we can see that there is a choice of which variables are optimized. There may be some motivation for keeping the ply ratios constant, or near constant. In which case, angle optimization will still give an efficient laminate. If angles are fixed, ply ratio optimization alone will also give an efficient laminate.

The capability to optimize hybrid laminates is easily added to the existing programs. When the A matrix is formed, the Q's associated with

the proper material are used. Also, the constraint test and gradient calculations must use the appropriate values of the G's for whichever material the given ply is made from. The example given in Table 5 shows the results for a hybrid made from alternating ply groups of graphite/epoxy and Kevlar/epoxy, with each orientation duplicated by both materials. For strength constraints, the Kevlar is usually completly removed. The combination of glass/epoxy and graphite/epoxy was found to give similar results. No strength advantage has been found by going to hybrid systems.

LOADS

N1= 4 MN/m
N2= 1 MN/m
N6= 0 MN/m

Material	Angle	# Plies Needed
Graphite	0	42.0
Kevlar	0	0.0
Graphite	90	4.9
Kevlar	90	0.0
Graphite	45	9.4
Kevlar	45	0.0
Graphite	-45	9.4
Kevlar	-45	0.0

TABLE 5: Hybrid Laminate Example

Potential Weight Savings

Optimization would be of little interest if the potential gains were only a few percent. In fact, for strength controlled laminates, the weight savings are usually in the range of 20-50%, as compared to quasi-isotropic lay-ups. The thickness difference due to optimization with a single biaxial load can be seen in Figure 20. This is a fairly general graph, since any biaxial load can be transformed to a shear-free axis (principle directions) and differences in N1 would just cause a proportional change in total thickness. It's interesting to note that the 0/90/45/-45 laminate is thinner than the 0/90. Beyond a load ratio of about 2 (N_1/N_2), the 90^o ply in the 0/90/+45/-45 laminate is dropped completly, making a tri-directional laminate that is more efficient than the 0/90. A good rule in design is to make the laminate axes and load principle axes coincide when there is a only a single load. The angle optimization routine will give this intuitive result. However, with 4 or more available orientations, the ply ratio optimization is forgiving if the principle directions are not used. A 0/90/45/-45 laminate was rotated as a rigid body with respect to a fixed 4:1 biaxial load. The laminate was optimized at 5^o increments of rotation. The difference beween the thickest and thinnest resulting laminate was only 5%.

When two or more independent loads are combined, the anisotropic advantage of composites becomes less significant, (because there is less of a distinct preferred direction) but the savings due to optimization can still be substantial. Because there are an infinite number of load combinations, it's impossible to draw any general graphs demonstrating the gains due to optimization. To give an indication of the trends, a series of 18 load combinations was devised, where each load combination

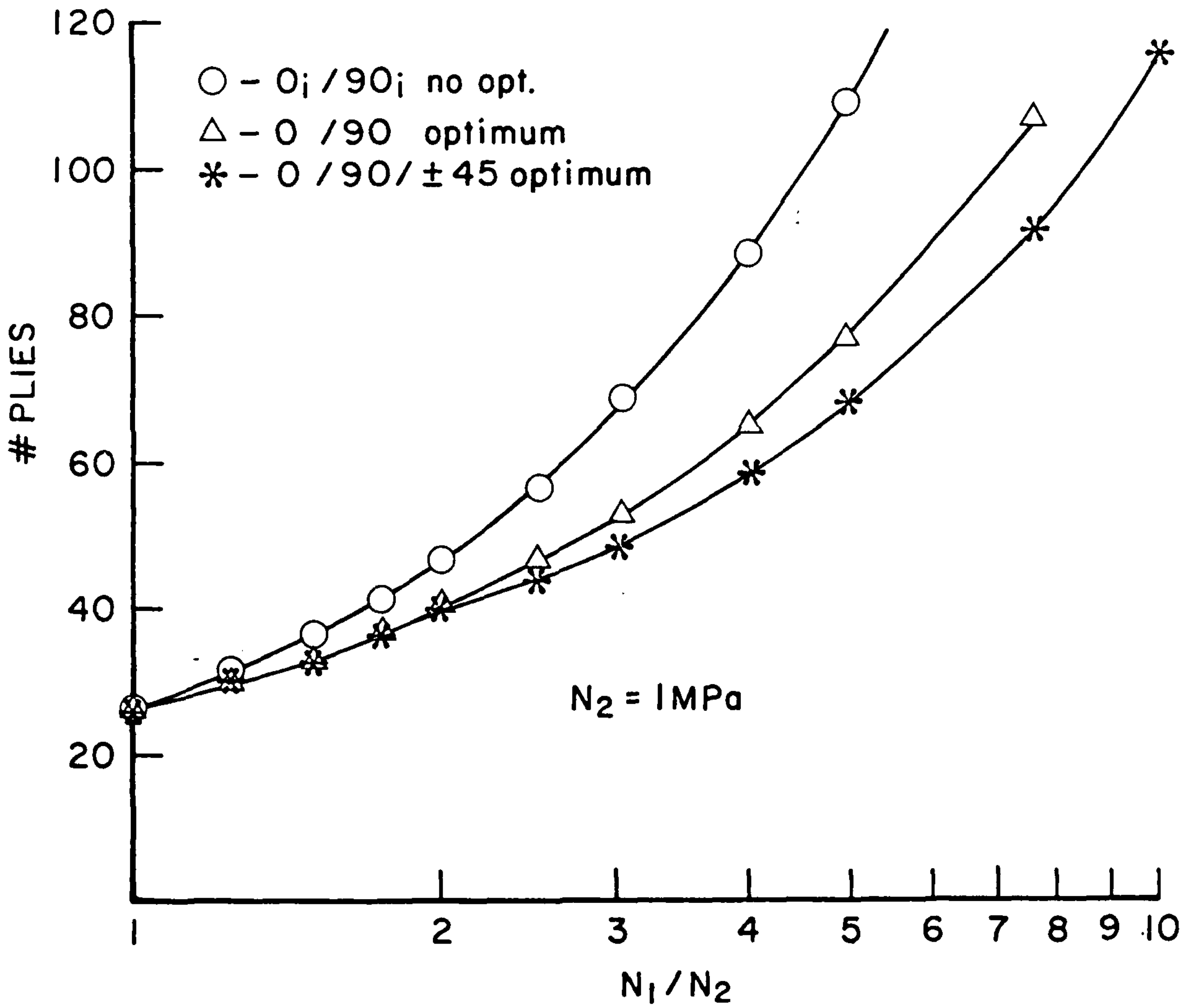
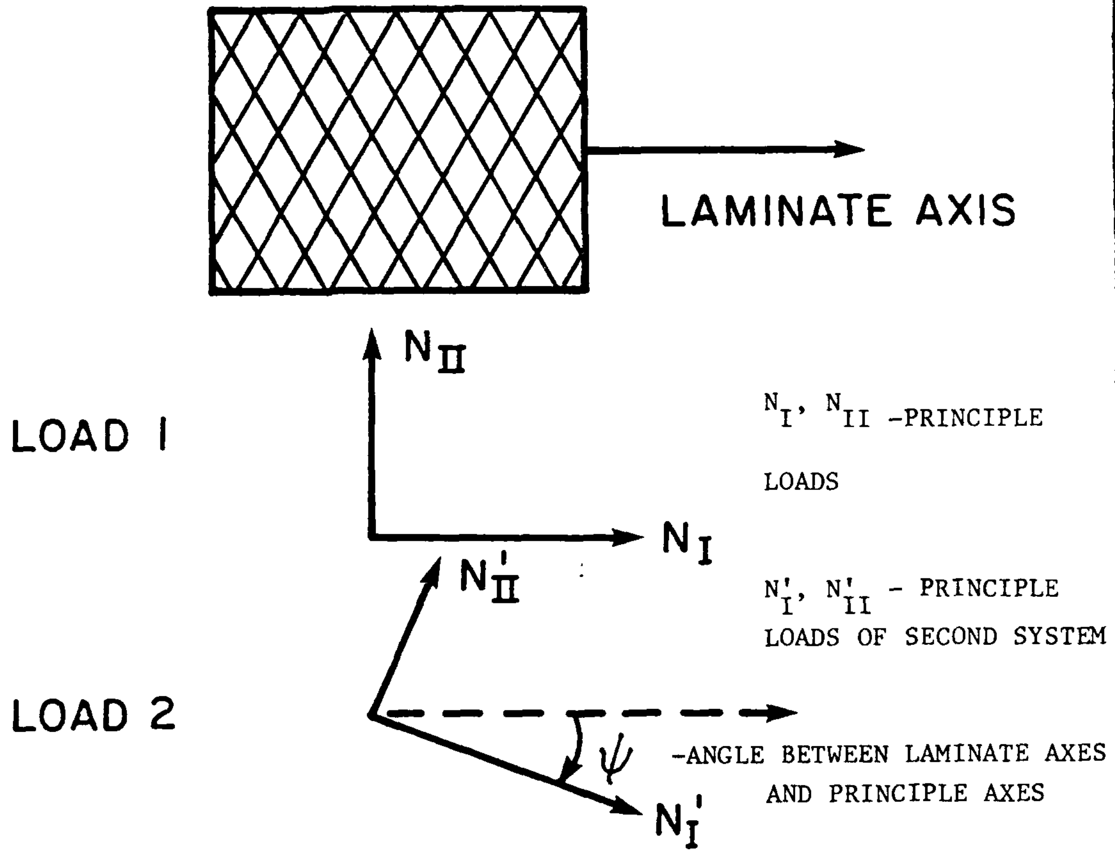


FIGURE 20: Totoal Number of Plies for Optimized and Equal Ratio Laminates Under a Single Load

consists of a pair of biaxial loads. Because of the directionality of composites, loads with differing principle axes are of greatest interest for excerising the procedure. The load combinations and principle axes orientations are given in Figure 21. This group of load cases is not intended to be all-encompassing, but represents some worst case conditions for taking advantage of a directional material. Most of the loads are in tension, although cases 13-15 are compression-compression and cases 16-18 are mixed tension and compression. The magnitudes of the principle components of the loads have been made equal in most of the cases in order to ensure both loads influence the final design. A small load might never form part of the boundary between feasible and infeasible design space. Intial angles are 0/90/45/-45 for all the types of optimization considered below. The next section will show that equal angular spacing is a good starting point for picking angles for the optimization code to work with. Ply ratio optimization alone will be considered first. Figure 22 shows the weight savings of optimized 0/90/45/-45 laminates versus unoptimized laminates of the same angles. Kevlar material was taken. Again, the load cases are arbitrary, but the point to be made is that around a 25% weight savings can be expected from using optimization with a wide variaty of loads. In some cases the savings can be even larger (40-50% for several of the load cases). To show that the results are not material dependent, the same loads have been applied to laminates made from graphite/epoxy (T300/5208). This time the savings are compared to aluminum, (with density difference included). The large differences between the optimized and unoptimized laminates are still evident (Figure 23).

The first 12 load cases (all tension-tension loads) were used to test the strain-sphere criterion. When averaged over the 12 loads, this



$N_I : N_{II}$	$N_I' : N_{II}'$	ψ = 20°	40°	60°
1:0	1:0	1	2	3
2:1	2:1	4	5	6
4:1	4:1	7	8	9
2:1	4:1	10	11	12
-2:-1	-2:-1	13	14	15
-2:-1	-2:1	16	17	18

LOAD CASE No.

FIGURE 21: Load Case Matrix for [illegible] Independent Loads

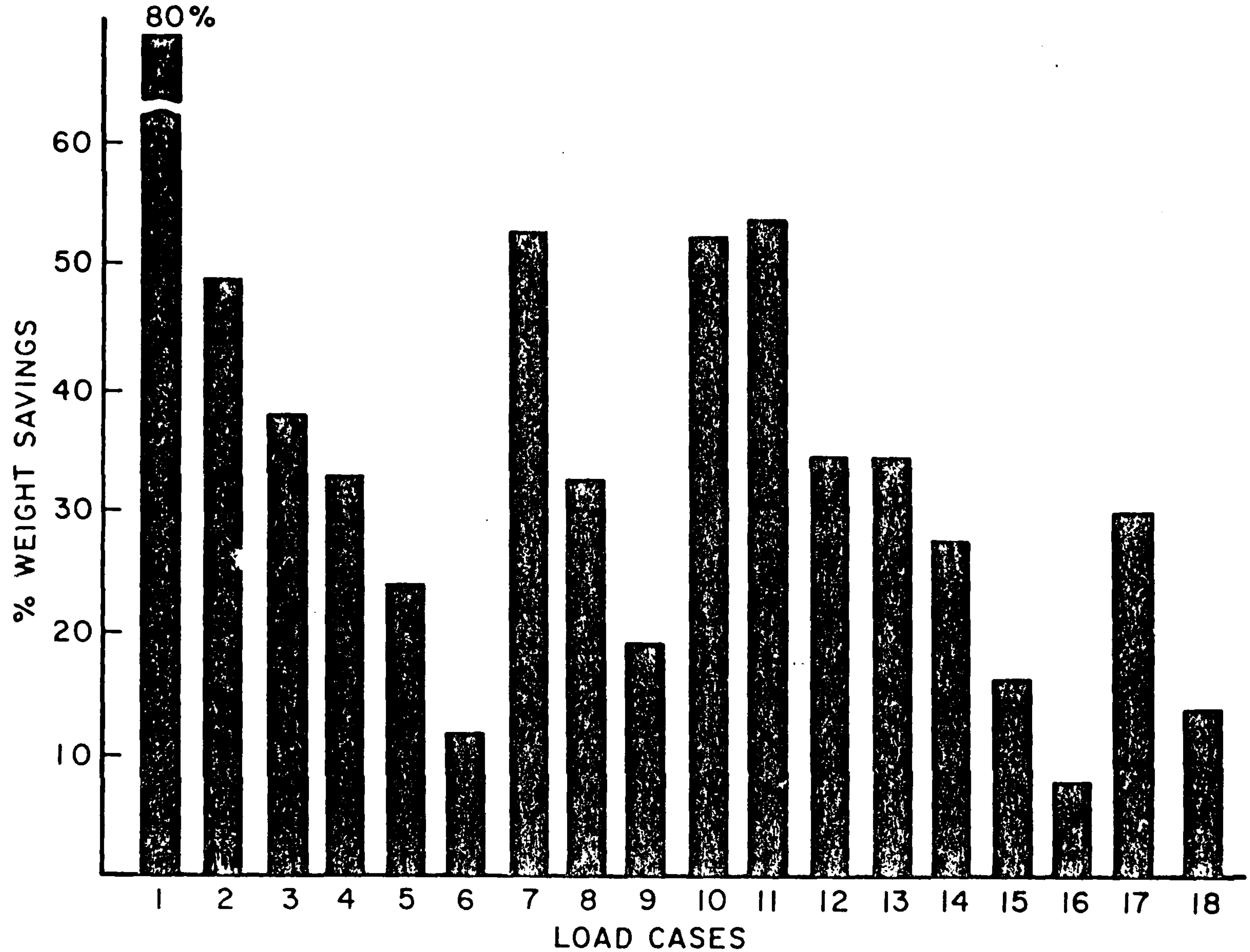


FIGURE 22: Weight Savings of Optimized Kevlar Laminates Over Quasi-Isotropic

Load cases correspond to Figure 21

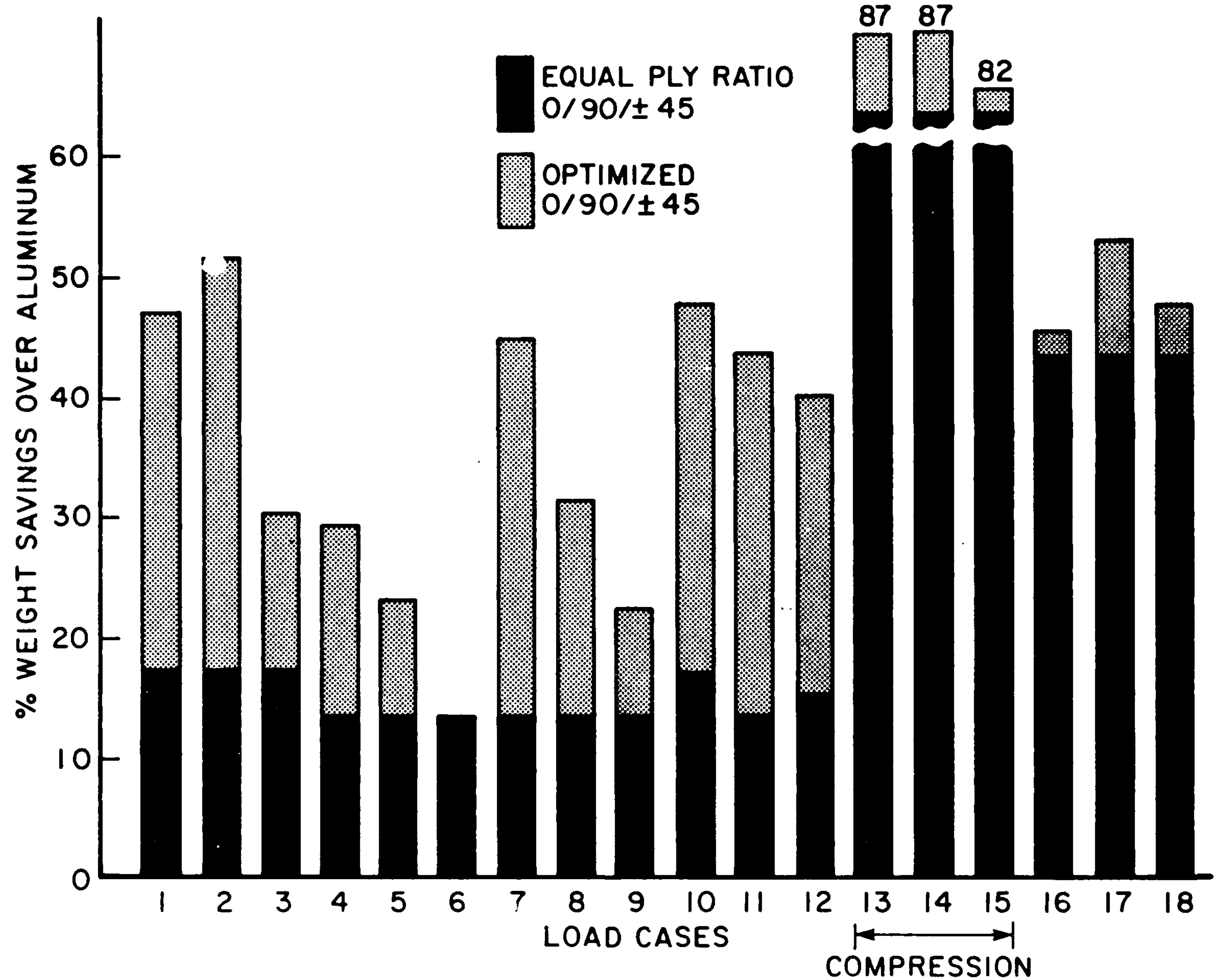


FIGURE 23: Weight Savings of Optimized and Equal Ratio T300/5208 Laminates Over Aluminum

Load cases correspond to Figure 21.

approximate criterion was found to be only 7% conservative as compared to the quadratic criterion. Thus, when only tension loads are considered (or with small compression components), the approximation may be desirable if computation time is a factor.

The orthotropic axis optimization is based on the strain-sphere criterion. This type of optimization was also tested against the first 12 load cases. The results are presented in Table 6. The average thickness is nearly the same as for ply ratio optimization alone, despite the conservative criterion and the added constraint of maintaining a balanced laminate.

Finally, angle optimization was also applied to laminates subjected to all 18 load cases, both with and without subsequent ply ratio optimization. A minimum angle change of 5 was always taken (see equation 31). With angle and ply ratio optimization, the average weight savings is about 6.5% better than ply ratio optimization alone, but the results for individual cases vary widely. Some load cases resulted in slightly greater thickness with angle optimization than without. The results are almost identical if angle optimization is used without ply ratio optimization at all. This demonstrates that the 2 types of design variables are almost redundant, and optimizing both is usually not required.

As Table 6 demonstrates, the designer has some options for picking the parameters to be optimized. The final results do not vary much for either ply ratio optimization with fixed orientations, orthotropic laminates (with rigid-body rotation allowed), or angle optimization alone. The degree of strength anisotropy appropriate to the design requirements can be achieved by varying any of these parameters. This means that composite materials have even more flexibility than

a) $\frac{\text{h orthotropic with rotation}}{\text{h ply ratio opt.}}$ b) $\frac{\text{h angle and ratio opt.}}{\text{h ply ratio opt.}}$

c) $\frac{\text{h angle opt.}}{\text{h ply ratio opt.}}$

LOAD CASE	a	b	c
1	1.12	0.75	0.75
2	1.41	0.94	0.94
3	0.99	0.79	0.79
4	1.03	1.00	1.01
5	1.02	0.97	0.98
6	0.91	0.94	0.94
7	1.05	1.10	1.19
8	0.97	0.88	0.92
9	0.91	0.90	0.90
10	1.02	0.97	0.98
11	0.99	0.96	0.96
12	0.96	0.94	1.03
13	----	1.04	1.05
14		1.12	1.12
15		0.92	0.95
16		0.79	0.84
17		0.98	1.01
18		0.83	0.91
Average	1.03	0.93	0.96

TABLE 6: Comparison of Alternate Optimization Parameters to Ply Ratio Optimization

previously imagined. The parameters to be optimized can be constrained by other considerations (such as manufacturing) and efficient laminates can still be produced.

Number of Angles Necessary

To use any of the methods described in this thesis, the number of ply orientations initially given to the optimization program must be chosen. The performance of various laminates with different numbers of initial angles was investigated to give some indication of how to pick these angles. A likely starting point for initial angles is to space the ply angles evenly over the 180 available. This class of laminates will be referred to as π/n laminates, where n is the number of orientations in the lamiante. A $\pi/4$ laminate has an angular spacing between ply groups of 45°. These lamiantes are quasi-isotropic for n greater than 2 [7]. This is a reasonable starting point for optimization since there are no preferred directions to initially bias the result.

The total thickness turns out to be almost independent of the number of angles for a single biaxial load (Table 7). By applying the 18 load cases introduced in the last section, a comparison for multiple loads can also be made. The average weight savings (compared to a $0_i/90_i/45_i/-45_i$ without optimization) is given in Table 8. For n greater than 3, the averages are very close. It is a little deceptive to take the average. When examined case-by-case, the thickness differences between the types of laminates can be great for a particular load case (Figure 24). These differnces may be largely due to numerical problems. With a large number of ply groups, the program may occasionally terminate early because of the large number of simultaneously active constraints. Despite this variation, the $\pi/4$ laminate seems to be adequate for multiple loads. Increasing the number of angles will not guarantee a

N1= 3 MN/m
N2= 1 MN/m
N6= 0 MN/m

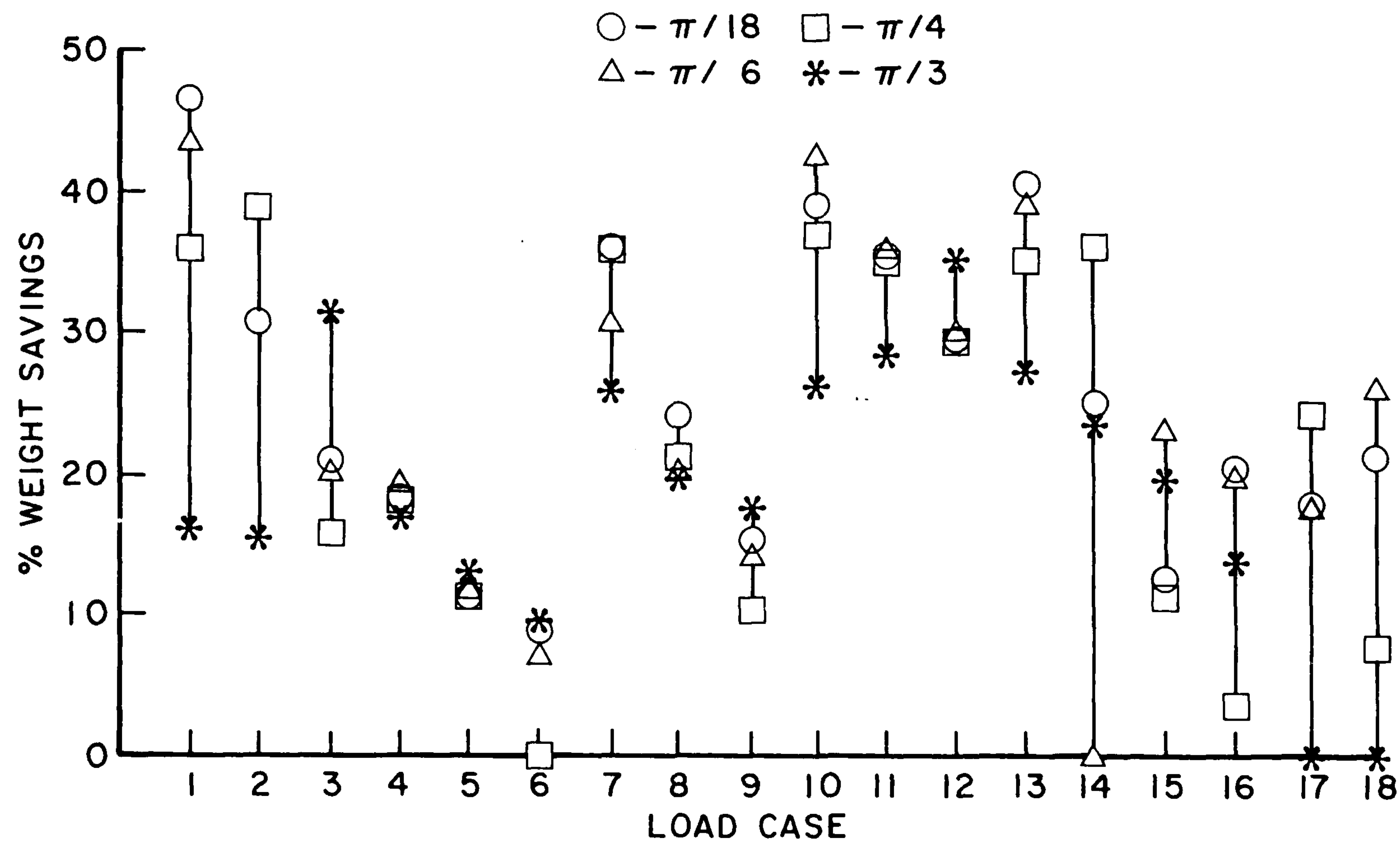
	# Ply Groups	Total # of Plies
60	3	52
45	4	49
30	6	51
18	10	50
10	18	51

Table 7: Total Thickness Required to Support a Single Load for Various /n Laminates

	#Ply Groups	% Weigth Savings
60	3	19
45	4	23
30	6	24
10	18	25

TABLE 8: Average Percent Weight Savings Over Quasi-Isotropic for All 18 Combined Load Cases

FIGURE 24: Weight Savings of Various π/n Laminates Over Quasi-Isotropic Load cases correspond to Figure 21. Material is T300/5208 .



better laminate.

The examples in this study included some $\pi/18$ laminates. An early idea was to find optimal angles by looking at a large number of initial angles and seeing what remained after ply ratio optimization. The actual result is a little surprising. Instead of a few optimal angles dominating the final laminate, the ply ratios plotted against angle form almost a continous function (Figure 25). All 18 ply groups are near failure for this laminate. For some multiple load test cases, 2 peaks in this pseudo-continous function would form. A case that formed more than 2 peaks was never found.

In conclusion, the number of initial angles can be bounded to a few choices. With only 2 orientations, we must have some way of picking the angles since the unoptimized laminate will have a directional preference. There doesn't seem to be any advantage to using more than 4 orientations. Thus, $\pi/4$ lamiantes were used for most of the examples in this thesis, and are suggested as a starting point for design.

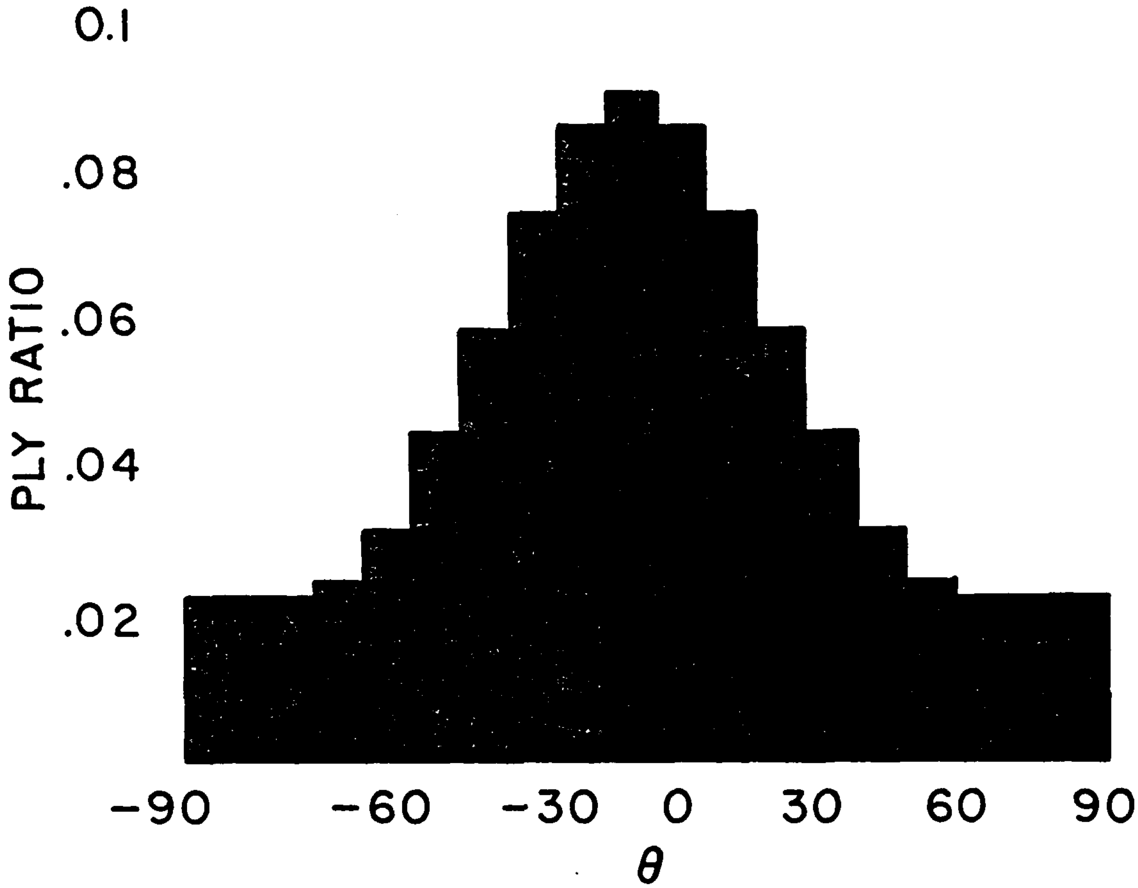


FIGURE 25: Ply Ratios Versus Angle for π/18 Laminate Optimized to a Single Load

($N_1 : N_2 = 2:1$)

IV. ANALYTIC STUDIES

Maximum Strain Energy Density

A visual representation of how a laminate adapts to the given load requirements would be desirable. A conventional failure envelope representation is not acceptable because with multiple loads, 3-dimensions would have to be shown in order to account for the differences in shear between the loads. The approach taken here is to plot the maximum strain energy density the laminate can sustain as a function of load principle axes orientation with respect to the laminate axes. Then, on the same graph, the strain energy density actually produced by various loads (in particular, the design loads) can also be plotted. There is a loss of information in such a graph. The combination of N_I to N_{II} (magnitutes of loads on the principle axes) that produces the maximum strain energy is an intermediate calculation and would not be displayed. The graph is not really a failure representation, since it would be possible to have loads which produced less strain energy but still caused failure. Despite these limitations, these graphs do give a good intuitive feel for the characteristics of an optimized laminate.

The approximate strain-sphere failure criterion is the starting point for the derivation. We assume the maximum strain energy occurs when the failure criterion reaches an equality. Then

$$\varepsilon_1^2 + \varepsilon_2^2 + \frac{1}{2}\varepsilon_6^2 = b^2 \qquad (33)$$

There are no shear loads, so that the stress-strain relation can be

written

$$\{\varepsilon\} = N_1[A^{-1}]\begin{Bmatrix} 1 \\ \lambda \\ 0 \end{Bmatrix} \tag{34}$$

where λ is defined by

$$\lambda = N_2/N_1 \tag{35}$$

The average, laminate strain energy density is given by

$$U = \frac{1}{2h}\{\varepsilon\}^T|A|\{\varepsilon\} \tag{36}$$

where h is the total thickness. Substituting equation (34) into (36) yields

$$\begin{aligned} U &= \frac{N_1^{\,2}}{2h}\left[A^{-1}\right]\begin{Bmatrix} 1 \\ \lambda \\ 0 \end{Bmatrix}^T [A]\,[A^{-1}]\begin{Bmatrix} 1 \\ \lambda \\ 0 \end{Bmatrix} \\ &= \frac{N_1^{\,2}}{2h}\begin{Bmatrix} 1 \\ \lambda \\ 0 \end{Bmatrix}[A^{-1}]\begin{Bmatrix} 1 \\ \lambda \\ 0 \end{Bmatrix} \\ &= \frac{N_1^{\,2}}{2h}(a_{11} + 2a_{12}\lambda + a_{22}\lambda^2) \end{aligned} \tag{37}$$

where a_{ij}'s are elements of the inverted A matrix.

Substituting equation (34) into the failure criterion yields

$$N_1^{\,2}\,[(a_{11} + a_{12}\lambda)^2 + (a_{12} + a_{22}\lambda)^2 + \frac{1}{2}(a_{13} + a_{23}\lambda)^2 = b^2$$

or

$$\begin{aligned} N_1^{\,2} = b^2/[(a_{11}^2 + a_{12}^{\,2} + \tfrac{1}{2}a_{13}^{\,2}) + (2a_{11}a_{12} \\ + 2a_{12}a_{22} + a_{13}a_{23})\lambda + (a_{12}^{\,2} + a_{22}^{\,2} + \tfrac{1}{2}a_{23}^{\,2})\lambda^2] \end{aligned} \tag{38}$$

Let

$$P = (a_{11}^{\,2} + a_{12}^{\,2} + \frac{1}{2}a_{13}^{\,2})$$

$$Q = 2(\frac{1}{2} a_{13}a_{23} + a_{11}a_{12} + a_{12}a_{22}) \tag{39}$$

$$R = (a_{12}{}^2 + a_{22}{}^2 + \frac{1}{2} a_{23}{}^2)$$

Then

$$N_1{}^2 = b^2/(P + Q\lambda + R\lambda^2) \tag{40}$$

Substituting (40) into (38), energy density becomes

$$U = \frac{b^2}{2h} \quad \frac{a_{11} + 2a_{12}\lambda + a_{22}\lambda^2}{P + Q\lambda + R\lambda^2} \tag{41}$$

A derivative with respect to λ is taken in order to find the maximum value.

$$\frac{dU}{d\lambda} = \frac{b^2}{2h} [(2a_{12} + 2a_{22}\lambda)(P + Q\lambda + R\lambda^2)$$

$$- (a_{11} + 2a_{12}\lambda + a_{22}\lambda^2)(Q + 2R\lambda)]/(P + Q\lambda + R\lambda^2)^2$$

If $du/d\lambda = 0$, then

$$(2a_{12} + 2a_{22}\lambda)(P + Q\lambda + R\lambda^2) - (a_{11} + 2a_{12}\lambda^2)(Q + 2R\lambda) = 0 \tag{42}$$

Let

$$\begin{aligned} A &= a_{22}Q - 2a_{12}R \\ B &= 2(a_{22}P - a_{11}R) \\ C &= 2a_{12}P - a_{11}Q \end{aligned} \tag{43}$$

By substituting (43) into (42), λ can be written as

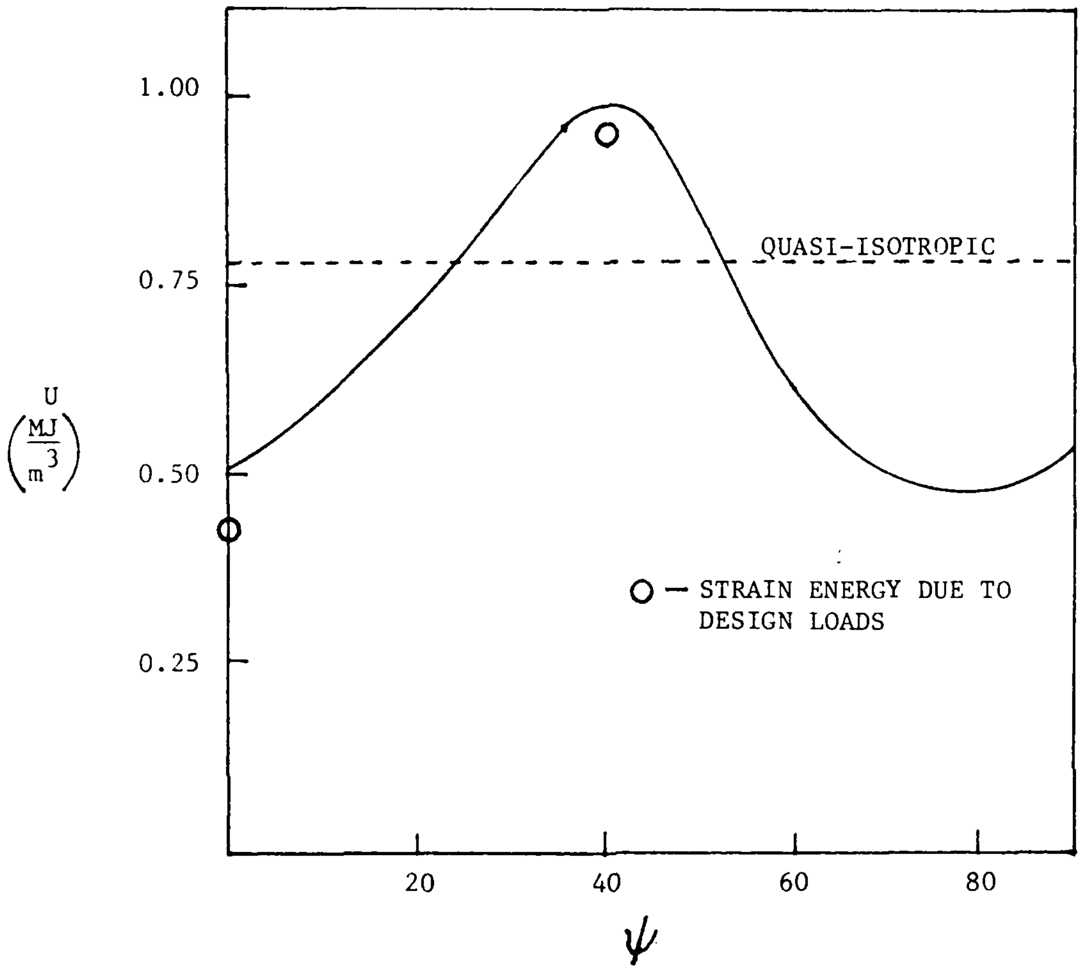
$$\lambda_{max} = \frac{-B \pm \quad B^2 - 4AC}{2A} \tag{44}$$

and the maximum strain energy density is given by

$$U_{max} = \frac{b^2}{2h} \frac{a_{11} + 2a_{12}\lambda_{max} + a_{22}\lambda^2_{max}}{P + Q\lambda_{max} + R\lambda^2_{max}} \tag{45}$$

To use the relations, a rigid body rotation is performed on the laminate, changing all the angles by the rigid body rotation angle Ψ . The A matrix and its inverse is calculated for the the new angles. Both values of λ are substituted into equation (45) and the one yielding the largest strain energy is taken.

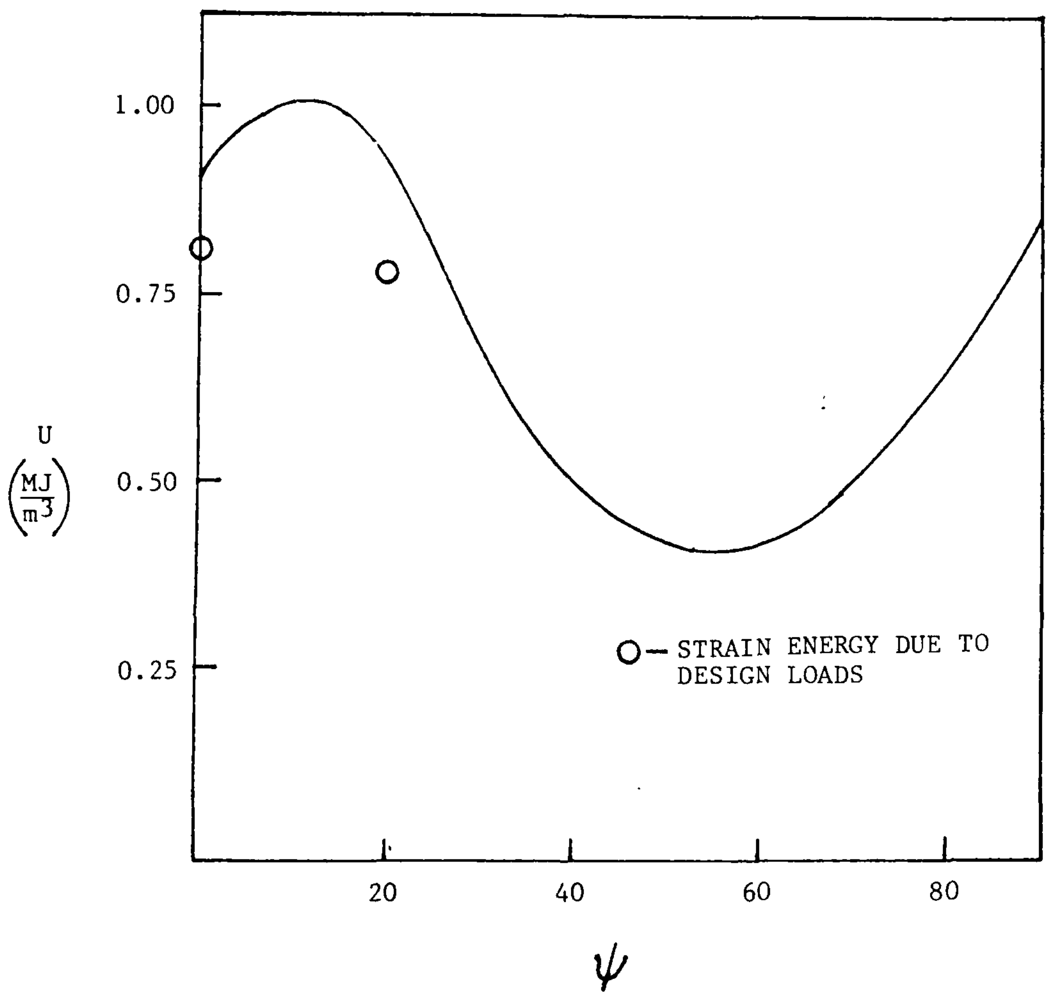
Figure 26 shows a typical graph for an optimized laminate. The strain energy density actually produced by the design loads are also plotted as points. We can see how the laminate has adapted to these loads. The function has to repeat after 90°because in the derivation, N_I and N_{II} are interchangeable. In Figures 27-29 the graphs are for laminates optimized to a pair of loads with equal principle magnitutes but with different angular spacings between their principle axes of the loads. The graphs show that as the angular spacing increases, the laminate's degree of aniostropy decreases. If there are many loads of near equal magnitude, and with widely spaced principle axes, then the laminate would have to be quasi-isotropic. There is a limit to how adaptable the laminate can be. The strain energy density will be close to a sin 4θ function, no matter how many ply groups are available.



Design Principle Loads and Orientations

$N_I = 2$ MN/m	$N'_I = 4$ MN/m
$N_{II} = 1$ MN/m	$N'_{II} = 1$ MN/m
$\psi = 0^\circ$	$\psi = 40^\circ$

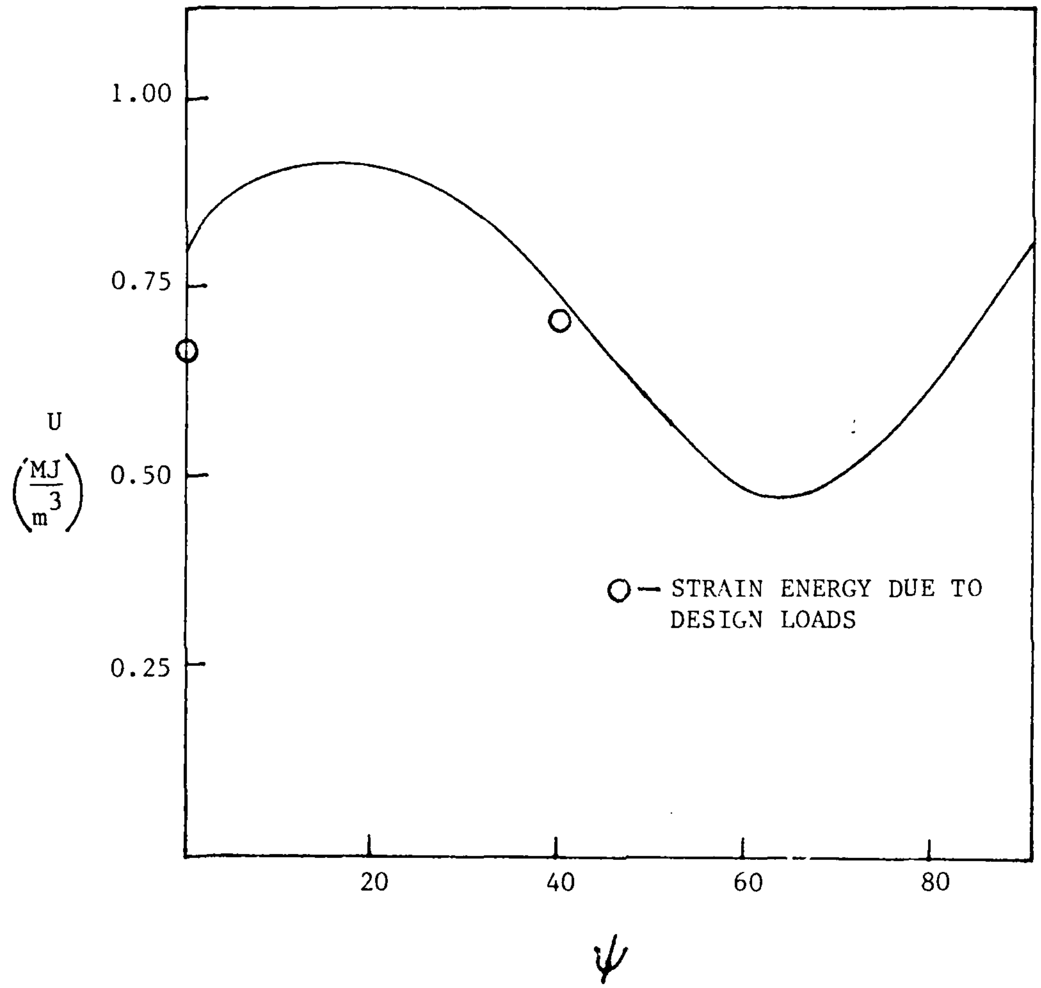
FIGURE 26: Strain Energy Density Versus Principle Direction



Design Principle Loads and Orientations

$N_I = 4$ MN/m	$N'_I = 4$ MN/m
$N_{II} = 1$ MN/m	$N'_{II} = 1$ MN/m
$\psi = 0^o$	$\psi = 20^o$

FIGURE 27: Strain Energy Density Versus Principle Direction



Design Principle Loads and Orientations

$N_I = 4$ MN/m $\quad$ $N'_I = 4$ MN/m

$N_{II} = 1$ MN/m $\quad$ $N'_{II} = 1$ MN/m

$\psi = 0^\circ$ $\quad$ $\psi = 40^\circ$

Figure 28: Strain Energy Density Versus Principle Direction

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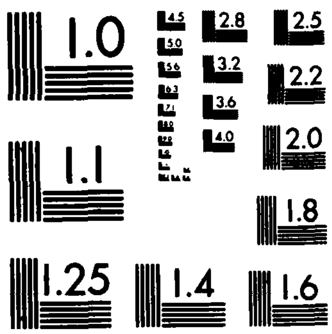
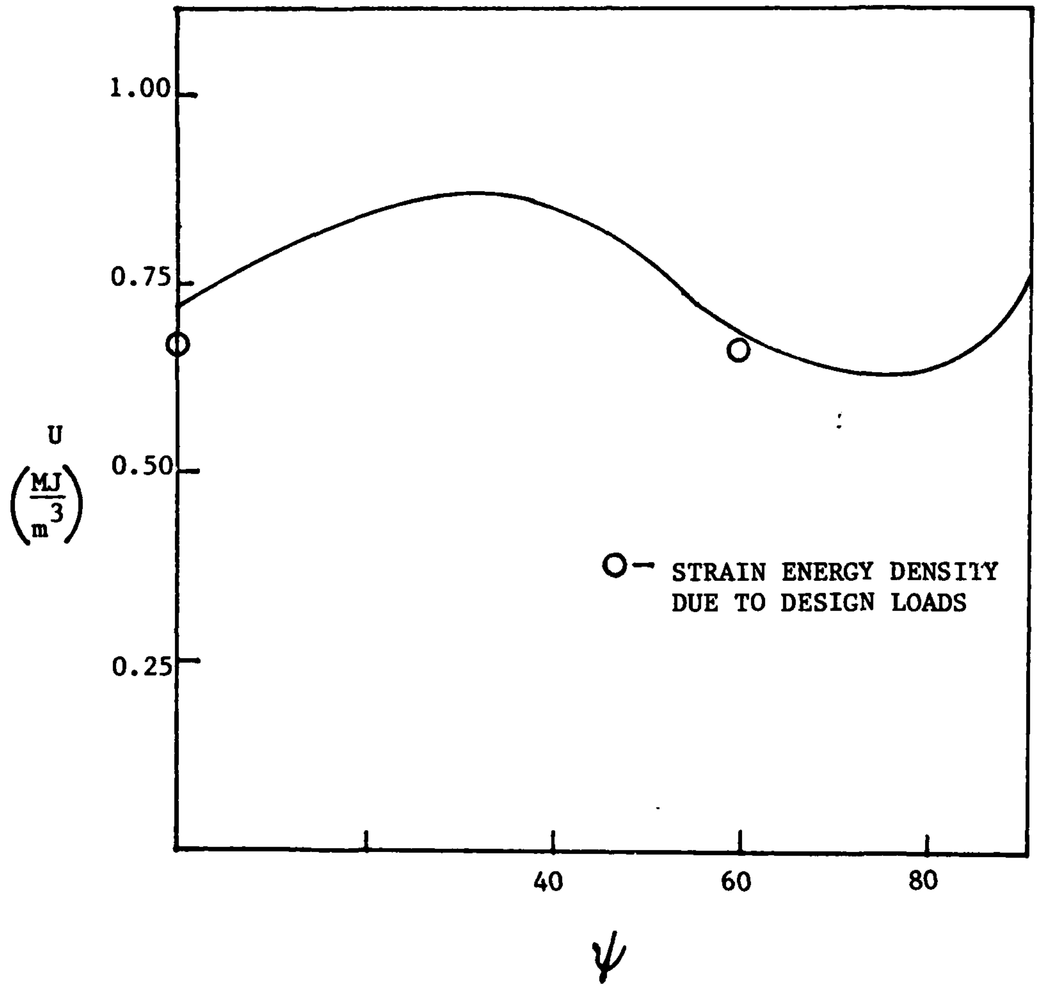


MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A



Design Principle Loads and Orientations

$N_I = 4$ MN/m $\quad$ $N'_I = 4$ MN/m

$N_{II} = 1$ MN/m $\quad$ $N'_{II} = 1$ MN/m

$\psi = 0°$ $\quad$ $\psi = 60°$

FIGURE 29: Strain Energy Density Versus Principle Direction

Optimality Criterion

The question of what constitutes an optimized laminate (besides the statement that it has minimum thickness) can be approached by considering what equality conditions must be true at the optimum. This is called an optimality criterion approach. Some existing optimization programs [3] are based on the assumption that strain energy density will be equal in all the plies at the optimum. This kind of criterion is based on experience with other types of structures, such as trusses. The failure criterion doesn't influence the selection of ply ratios, but only the total thickness scaling.

The strain-sphere criterion is simple enough that for single loading conditions, an optimality criterion can be derived directly from the failure equation. Taking only ply group thickness as the design variables, the minimum thickness point can be found from the Langrange multiplier equation

$$\vec{\nabla}h + \lambda\vec{\nabla}C = 0 \tag{46}$$

Terms of the gradient of the constraint can be written as

$$\frac{\partial C}{\partial h_i} = 2\vec{\varepsilon}\ |T|\vec{\varepsilon},_{h_i} \tag{47}$$

where

$$T = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{vmatrix} \tag{48}$$

From equation (21), we have the result that

$$\vec{\varepsilon},_{h_i} = -|A^{-1}|\ |Q^{(i)}|\ \vec{\varepsilon} \tag{49}$$

Substituting into the Langrange multiplier equation (46), for each component, we have

$$1 + 2\lambda\vec{\varepsilon}^{T} \, |T| \, \vec{\varepsilon}_{,h_i} = 0 \tag{50}$$

Thus, each ply group must satisfy the equation

$$\vec{\varepsilon}^{T}|T| \; |A^{-1}| \; |Q^{(i)}| \; \vec{\varepsilon} = \tilde{\lambda} \tag{51}$$

where $\tilde{\lambda}$ is the same constant for each ply group.

The strain energy density criterion could be written as

$$\vec{\varepsilon}^{T}|Q^{(i)}| \; \vec{\varepsilon} = \lambda \tag{52}$$

which, again, must be satisfied for each ply group. There is a significant difference between the two criteria. The implications of equation (51) should be studied in more detail. Perhaps a more direct solution to the optimization problem can be found.

V. CONCLUSIONS

A series of effective laminate optimization programs have been developed and thoroughly tested. The programs have been designed to be compact and efficient enough to operate on the some of the smallest microcomputers. Although not as general or sophisticated as some of the optimization codes currently available, these programs offer good performance and are very easy to use even for those unversed in optimization. No program in the literature has been found that can perform angle optimization or the orthotropic axis optimization. Thus, much greater flexibilty is now available to the designer.

The gains due to optimization have been found to be substantial, with typically a 30% weight savings as compared to quasi-isotropic laminates. Surprisingly, these large gains can be made with either of a couple of design parameters. The designer can either optimize the ply ratios, or the angles and usually get equally efficient laminates. Or, he may chose to constrain the laminate to be orthotropic after optimization. If the orthotropic axis is free to change, efficient laminates can be designed.

By trying many example cases, it has been found that a $\pi/4$ laminate is a good starting laminate. By starting with quasi-istropic laminates, no knowledge of desired starting orientations for the particular loads is needed. Increasing the number of initial orientations does not seem to improve the final laminates.

An approximate failure criteria has been found to give good results while substantially decreasing the computation times needed. The approximate criteria could be particularly important when the optimization procedure is tied into a finite element code on an

iterative basis, where the repeated optimizations could become excessively time consuming.

The approximate criteria also allows some analytic studies of optimized laminates. A representation of the optimized laminates strength anisotropy has been developed based on the maximum strain energy density. Graphs made with this formulation show how the laminates match the load requirements. Also, there is a limit to the adaptablility of a laminate. As more load requirement are added, eventually the laminate must become quasi-isotropic. An optimality criterion can also be derived from the approximate failure criterion which can be the subject of future investigations.

Hopefully, tailored laminates will come more common as these new tools are made available to designers, enhancing the desirability of composites.

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APPENDIX A

Angular Derivatives

The derivatives of the stiffness and failure parameters are found using the multiple angle transformations of Tsai [5]

$$\frac{\partial Q_{11}}{\partial \theta} = -2U_2 \sin 2\theta - 4U_3 \sin 4\theta$$

$$\frac{\partial Q_{22}}{\partial \theta} = 2U_2 \sin 2\theta - 4U_3 \sin 4\theta$$

$$\frac{\partial Q_{12}}{\partial \theta} = 4U_3 \sin 4\theta$$

$$\frac{\partial Q_{66}}{\partial \theta} = 4U_3 \sin 4\theta$$

$$\frac{\partial Q_{16}}{\partial \theta} = U_2 \cos 2\theta + 4U_3 \cos 4\theta$$

$$\frac{\partial Q_{26}}{\partial \theta} = U_2 \cos 2\theta - 4U_3 \cos 4\theta$$

where

$$U_2 = 1/2\ (Q_{xx} - Q_{yy})$$
$$U_3 = 1/8\ (Q_{xx} + Q_{yy} - 2Q_{xy} - 4Q_{ss})$$

Partials of G_{ij} can be found with the same equations, but with

$$U_2 = 1/2\ (G_{xx} - G_{yy})$$

$$U_3 = 1/8\ (G_{xx} + G_{yy} - 2G_{xy} - 4G_{ss})$$

The linear terms of the failure equation has become

$$\frac{\partial G_1}{\partial \theta} = -2q \sin 2\theta$$

$$\frac{\partial G_2}{\partial \theta} = 2q \sin 2\theta$$

$$\frac{\partial G}{\partial \theta} = 2Q \cos 2\theta$$

where

$q = 1/2\ (G_x - Gy)$

APPENDIX B

Program for Ply Ratio and Angle Optimization

The following program optimizes composite laminates for minimum weight subject to inplane strength requirements. Program options are: 1) optimized ply ratios 2) optimize ply angles and ratios 3) perform laminate plate analysis without optimization. Inputs include initial ply angles, loads (multiple independent loads possible) and a material selection. Material properties for common composites are stored in a library, or new properties can be entered by following prompts. The program is interactive and use should be obvious from displayed prompts. A typical computer/user dialogue is given below, along with the resulting output.

The program is written for an Epson HX-20 microcomputer which uses a fairly standard form of BASIC. The major exception are the GET% and PUT% commands to addrss the material library. These can be replaced by disk file operations on most other computers. The other possible change would be the explicit double precision symbol "#" used in the program. Test have shown that double precision is not really needed and could be left out when using other machines.

COMPUTER/USER DIALOGUE

LCD Display	Keyboard Response (comments in parenthesis)
Press any key when desired Material appears	RUN RETURN (unless otherwise noted, "Return" key pressed after each keyboard entry)
T300/5208	
B(4)/5505	
AS/3501	
Scotchply 1002	
Kevlar 49/Epoxy	
Aluminum	
New	(random key pressed when "New" appears on screen)
REVIEW OR NEW DATA (R/N) ?	N
WHICH MATERIAL WILL YOU REPLACE (0-5) ?	5 (materials numbered in same order as listed T300/5208=0)
EX(GPa) = ?	185
EY(GPa) = ?	6.76
VX = ?	.2
ES(GPa) = ?	5.86
X(MPa) = ?	680
X'(MPa) = ?	690 (primed constants imply compressive properties)
Y(MPa) = ?	16
Y'(MPa) = ?	186
S(MPa) = ?	72
THICKNESS (m.) = ?	125E-6 (ply thickness)
NAME (15 CHR MAX) ?	HMS/3002M
ADDITIONAL CHANGES ?	N

LCD Display	Keyboard Response (comments in parenthesis)
. . . .	(Materials list begins again, this time with the new material replacing aluminum, when it appears a key is pressed)
HOW MANY PLY GROUPS ?	4
ENTER PLY GROUP	
ORIENTATIONS	
PLY 1 = ?	0
PLY 2 = ?	90
PLY 3 = ?	45
PLY 4 = ?	-45
ENTER NUMBER OF INDEPENDENT LOADING CONDITIONS ?	2
LOAD 1 in MPa	
N1 = ?	3
N2 = ?	2
N6 = ?	.5
LOAD 2 in MPa	
N1 = ?	1
N2 = ?	4
N6 = ?	0
OPTIMIZATION OR ANALYSIS (O/A)	O
RATION OR ANGLE OPTIMIZATION (R/A)	R

WORKING ITERATION 1 TOTAL THICKNESS = 1.71342 E-02 m. 137.07 PLIES HIT ANY KEY TO CONTINUE	(after 4 iterations and about 7 minutes the computer beeps that the solution has been found. This example ran for an unusually long time. Most problems will run in less time)
Press Y if printout of displayed result is desired. Press N if not	(press any key, no return)
PLY PROPERTIES	Y (return key not used for these responses)
LOADS	Y
TOTAL THICKNESS & PLY RATIOS	Y
STRENGTH	Y
LAMINATE STRAINS	Y
STIFFNESS MATRIX	Y
COMPLIANCE MATRIX	Y
PLY RATIO GRAPH	Y (after entire list of printout options is presented, computer produces the printout shown on next page)
FINISHED HIT ANY KEY TO CONTINUE	(pressing a key restarts program. Press "BREAK" key to exit).

[redacted]
Material Properties
HMS/3002M
EX= 185 GPa
EY= 6.76 GPa
ES= 5.86 GPa
VX= .2
X= 680 MPa
X'= 690 MPa
Y= 16 MPa
Y'= 186 MPa
S= 72 MPa
Ply Thickness .000125 m

LOADING 1
N 1= 3 MN/m
N 2= 2 MN/m
N 6= .5 MN/m
LOADING 2
N 1= 1 MN/m
N 2= 4 MN/m
N 6= 0 MN/m

Total thickness=
.0171E+00 m.
137.07 Plies

ANGLE	RATIO	#PLIES
0	.3476	47.65
90	.5281	72.38
45	.1243	17.04
-45	0	0

STRENGTH RATIOS
1=ULTIMATE STRAIN
>1 IS SAFE
LOADING 1

PLY	
0	1.1622
90	1
45	1.1616

LOADING 2

PLY	
0	1.0053
90	1.2115
45	1.0122

LAMINATE STRAINS
LOADING 1
e1=+2.182E-03
e2=+0.902E-03
e6=+1.096E-03
LOADING 2
e1=+0.697E-03
e2=+2.216E-03
e6=-1.467E-03

Norm. |A| in GPa.

74.762	6.510	5.548
6.510	106.967	5.548
5.548	5.548	11.016

Compliance (normalized)
in 1/TPa.

13.921	-0.497	-6.760
-0.497	9.617	-4.593
-6.760	-4.593	96.497

ENGINEERING CONSTANTS

E1= 71.8 GPa
E2=104.0 GPa
E6= 10.4 GPa
v21= 0.036
v61=-0.486
v16=-0.070

Output Produced from Example Dialogue

```
10 '** MAIN CLASS**
20 CLEAR 75,330
30 WIDTH 20,4
40 DEFFIL 55,0
50 DEFINT I-P:DEFDBL F
60 DIM A(3,3),B(6,9),C(6
,6),D(3,3),G(3,3),XN(4,3
),AI(3,3),Q(3,3),H(6),R(
3),S(3),T(6),U(5),V(7),X
(6),Y(3),Z(6),E(4,3)
65 DIM W(24,6),CON(24)
70 DIM C%(10,2)
80 DEF FNDEG(X)=X*57.295
78
90 DEF FNRAD(X)=X/57.295
78
100 '** MAIN **
105 RESTORE
110 READ IMAX,E2,E5,E6
120 ITER=1
130 GOSUB 2540
140 CLS:PRINT "OPTIMIZAT
ION OR":INPUT "ANALYSIS
(O/A)";A$
150 IF A$="A" THEN 6500
152 INPUT"RATIO OR ANGLE
 OPT  (R/A)";A$:IF A$="A
" THEN INPUT "DELTA";DEL
TA
155 CLS: PRINT"WORKING":
PRINT"ITERATION";ITER
170 GOSUB 2990
180 GOSUB 2330
190 GOSUB 2190
196 IF A$<>"A" THEN 200
197 DELTA=FNRAD(DELTA):G
OSUB 10000
200 GOSUB 1680
205 CLS: PRINT"WORKING":
PRINT"ITERATION";ITER
210 IF F$="FAIL" THEN 33
00
220 GOSUB 1370
230 ITER=ITER+1
240 IF F$="FAIL" OR ITER
>IMAX THEN 3300
250 GOTO 200
260 '** CONSTRAINT TEST*
*
270 G$="PASS": NC=0
280 FOR P=1 TO NPLY
290 IF H(P)=0 THEN 445
300 II=P :GOSUB 1230
310 FOR N=1 TO NL
320 FCON=-1
330 FOR K=1 TO 3
340 FOR J=1 TO 3
350 FCON=FCON+G(K,J)*E(N
,J)*E(N,K)
360 NEXT J
370 FCON=FCON+S(K)*E(N,K
)
380 NEXT K
390 IF FCON>0 THEN G$="F
AIL": RETURN
400 IF FCON<-E5 THEN 440
```

Comments

20-40 commands to configure the machine

50 Implicit integer and double precision

80-90 convert radians to degrees and degrees to radians

130 - Gosub input

170 - Gosub invariants

180 - Gosub transformations

190 - Gosub initial feasible pt.

200 - Gosub direction

220 - Gosub new thickness

290 - if ply thickness zero, ignore constraint

300 - Get G matrix for ply being tested

320 - 380 Solve FCON = $G_{ij}\varepsilon_i\varepsilon_j + G_i\varepsilon_i - 1$

410 - 430 If FCON is close to zero identify constraint as active, make list in C% and increment constraint counter

```
410 NC=NC+1
420 C%(NC,1)=P
430 C%(NC,2)=N
440 NEXT N
445 NEXT P
450 RETURN
456 STOP
460 '** GRADIENT **
475 UNORM=0
480 II=P: GOSUB 1230
490 FOR L=1 TO NPLY
500 IF H(L)=0 THEN 700
510 II=L: GOSUB 1120
520 FOR J=1 TO 3
530 R(J)=0
540 FOR K=1 TO 3
550 R(J)=R(J)-Q(J,K)*E(N
,K)
560 NEXT K,J
570 FOR J=1 TO 3
580 Y(J)=0
590 FOR K=1 TO 3
600 Y(J)=Y(J)+AI(J,K)*R(
K)
610 NEXT K,J
620 Z(L)=0
630 FOR J=1 TO 3
640 FOR K=1 TO 3
650 Z(L)=Z(L)+G(J,K)*(Y(
J)*E(N,K)+E(N,J)*Y(K))
660 NEXT K
670 Z(L)=Z(L)+S(J)*Y(J)
680 NEXT J
690 UNORM=UNORM+Z(L)*Z(L
)
700 NEXT L
710 UNORM=SQR(UNORM)
720 FOR L=1 TO NPLY
730 Z(L)=Z(L)/UNORM
740 NEXT L
760 RETURN
770 '** STRAINS **
780 DIM F(3,3)
790 FOR I=1 TO 3
800 FOR J=1 TO 3
810 F(I,J)=A(I,J)+D(I,J)
*S
820 NEXT J,I
830 DET#=F(1,1)*F(2,2)*F
(3,3)+2*F(1,2)*F(2,3)*F(
1,3)-F(2,2)*F(1,3)*F(1,3
)-F(1,1)*F(2,3)*F(2,3)-F
(3,3)*F(1,2)*F(1,2)
840 AI(1,1)=(F(2,2)*F(3,
3)-F(2,3)*F(2,3))/DET#
850 AI(2,2)=(F(1,1)*F(3,
3)-F(1,3)*F(1,3))/DET#
860 AI(1,2)=(F(1,3)*F(2,
3)-F(1,2)*F(3,3))/DET#
870 AI(3,3)=(F(1,1)*F(2,
2)-F(1,2)*F(1,2))/DET#
880 AI(1,3)=(F(1,2)*F(2,
3)-F(2,2)*F(1,3))/DET#
890 AI(2,3)=(F(1,2)*F(1,
3)-F(1,1)*F(2,3))/DET#
900 AI(2,1)=AI(1,2):AI(3
,2)=AI(2,3):AI(3,1)=AI(1
,3)
```

480 - Get G matrix for designated ply

510 - For each ply, get Q matrix

540 - 560 $\vec{R} = -\frac{\partial}{\partial h} A \vec{\varepsilon}$

580 - 610 $\vec{Y} = |A^{-1}| \vec{R}$

$$Y = \frac{\partial}{\partial h_k} \vec{\varepsilon}$$

620 - 680 $\vec{\nabla}$ (FCON) $= [G_{ij}(\varepsilon_i \frac{\partial \varepsilon j}{\partial h_k} + \frac{\partial \varepsilon i}{\partial h_k} \varepsilon_j) + G_i (\frac{\partial \varepsilon i}{\partial h_k})] \hat{h}k$

690 - 730 Normalize $\vec{\nabla}$(FCON)

790 - 820 "F" is the A matrix corresponding to a point S along the Z vector

830 - 900 invert A

920 - 970 Solve $\vec{\varepsilon}=|A^{-1}| \vec{N}$ for each independent loading

```
910 ERASE F
920 FOR I=1 TO NL
930 FOR J=1 TO 3
940 E(I,J)=0
950 FOR K=1 TO 3
960 E(I,J)=E(I,J)+AI(J,K
)*XN(I,K)
970 NEXT K,J,I
980 RETURN
990 '** A MATRIX **
1000 FOR I= 1 TO 3
1010 FOR J=1 TO 3
1020 A(I,J)=0: D(I,J)=0
1030 NEXT J,I
1040 FOR I=1 TO NPLY
1050 II=I: GOSUB 1120
1060 FOR J=1 TO 3
1070 FOR K=1 TO 3
1080 A(J,K)=A(J,K)+Q(J,K
)*H(I)
1090 D(J,K)=D(J,K)+Q(J,K
)*Z(I)
1100 NEXT K,J,I
1110 RETURN
```

1000 - 1100 The matrix D is formed so that along the Z vector

$$|A| = |\tilde{A}| + |D| \cdot S$$

where S is a scalor

```
1120 '** FORM Q **
1130 Q(1,1)=C(II,1)
1140 Q(1,2)=C(II,3)
1150 Q(1,3)=C(II,5)
1170 Q(3,1)=C(II,5)
1180 Q(3,2)=C(II,6)
1190 Q(3,3)=C(II,4)
1195 Q(2,3)=C(II,6)
1200 Q(2,2)=C(II,2)
1210 Q(2,1)=C(II,3)
1220 RETURN
```

1130 - 1210 Convert C array into 3 x 3 Q matrix for ply designated by II

```
1230 '** FORM G **
1240 G(1,1)=B(II,1)
1250 G(1,2)=B(II,3)
1260 G(1,3)=B(II,5)
1270 G(2,1)=B(II,3)
1280 G(2,2)=B(II,2)
1290 G(2,3)=B(II,6)
1300 G(3,1)=B(II,5)
1310 G(3,2)=B(II,6)
1320 G(3,3)=B(II,4)
1330 S(1)=B(II,7)
1340 S(2)=B(II,8)
1350 S(3)=B(II,9)
1360 RETURN
```

1240 - 1350 Convert B array into 3 x 3 G matrix for ply designated by II. Linear failure terms placed in vector S

```
1370 '** NEW H VECTOR **
1380 SMAX=1E10
1390 FOR I=1 TO NPLY
1400 IF Z(I)<>0 THEN S=-
H(I)/Z(I)
1410 IF S>0 AND S<SMAX T
HEN SMAX=S
1420 NEXT I
1430 F$=""
1440 IF SMAX> 10 THEN F$
="FAIL": RETURN
1450 S1=0: S2=SMAX: S=SM
AX
1460 IF NC=0 THEN 1590
1470 GOSUB 770: GOSUB 26
0
1480 IF G$="FAIL" THEN S
2=S ELSE S1=S
1490 IF S1=SMAX THEN 153
5
```

1380 - 1420 Find distance along $\vec{Z}$ to find $h_i = 0$ constraint

1450 - 1500 Bisection method to find distance to next constraint. If no constraints violated at S = SMAX then stop search

```
1500 S=(S1+S2)/2
1510 IF S2-S1<E2 AND S1=
0 THEN F$="FAIL": S=0: G
OTO 1650
1520 IF S1/(S2-S1)<4 THE
N 1470
1530 S=S/2
1535 SREF=0
1540 FOR I=1 TO NPLY
1550 H(I)=H(I)+Z(I)*S
1560 IF H(I)<E2 THEN H(I
)=0
1570 SREF=SREF+H(I)*H(I)
1580 NEXT I
1590 S=0:SREF=SQR(SREF)
1600 GOSUB 990: GOSUB 77
0: GOSUB 2020
1610 IF SREF-S<E2 THEN F
$="FAIL"
1620 FOR I=1 TO NPLY
1630 H(I)=H(I)*S/SREF
1640 NEXT I
1650 S=0
1660 GOSUB 990: GOSUB 77
0: GOSUB 260
1670 RETURN
1680 '** DIRECTION **
1690 Z=0: UNORM=1
1700 FOR I=1 TO NPLY
1710 X(I)=0
1720 Z=Z+SGN(H(I))
1730 NEXT I
1740 Z=1/SQR(Z)
1750 IF NC=0 THEN 1860
1760 FOR I=1 TO NC
1770 P=C%(I,1): N=C%(I,2
)
1780 GOSUB 460
1790 FOR J=1 TO NPLY
1800 LET X(J)=X(J)-Z(J)
1810 NEXT J,I
1815 UNORM=0
1820 FOR J=1 TO NPLY
1830 UNORM=UNORM+X(J)*X(
J)
1840 NEXT J
1850 UNORM=SQR(UNORM): T
EST=0
1860 FOR I=1 TO NPLY
1870 X(I)=X(I)/UNORM
1880 TEST=TEST+X(I)*Z*SG
N(H(I))
1890 NEXT I
1900 UNORM=0
1910 FOR I=1 TO NPLY
1920 Z(I)=X(I)-TEST*Z*SG
N(H(I))
1930 UNORM=UNORM+Z(I)*Z(
I)
1940 NEXT I
1950 IF UNORM<1E-6 THEN
F$="FAIL": RETURN  ELSE
F$=""
1960 UNORM=SQR(UNORM)
1970 FOR I=1 TO NPLY
1980 Z(I)=Z(I)/UNORM
1990 NEXT I
2000 GOSUB 990
2010 RETURN
```

1530 - 1600 at point halfway between constraints, use strain ratio routine to find how much the laminate thickness can be reduced

1610 If change in thickness small, set flag to halt program

1620 - 1660 Update h vector, A matrix, strains

1760 - 1840 For each active constraint call gradient subroutine. Sum negative of each gradient into $\vec{X}$ and normalize $\vec{X}$

1860 - 1890 Take dot product of $\vec{X}$ and unit normal to Σhi = const. plane

1910 - 1940 $\vec{Z}$ is a vector parallel to the Σhi = const. plane and pointing away from the active constraints

1950 if the magnitude of $\vec{Z}$ is very small, a local minima has been reached

```
2020 '** STRAIN RATIO **
2030 FOR P=1 TO NPLY
2040 IF H(P)=0 THEN 2160
2050 II=P: GOSUB 1230
2060 FOR N=1 TO NL
2070 B#=0:C#=0
2080 FOR I=1 TO 3
2090 FOR J=1 TO 3
2100 C#=C#-SREF*SREF*G(I
,J)*E(N,I)*E(N,J)
2110 NEXT J
2120 B#=B#-SREF*S(I)*E(N
,I)
2130 NEXT I
2140 SVAL=(-B#+SQR (B#*B
#-4*C#*(1-E6)))/(2*(1-E6
))
2150 IF SVAL>S THEN S=SV
AL
2155 NEXT N
2160 NEXT P
2180 RETURN
2190 '** IFP **
2200 Z=1/SQR(NPLY)
2210 FOR I=1 TO NPLY
2220 Z(I)=Z: H(I)=Z
2230 NEXT I
2240 GOSUB 990
2250 S=0: SREF=1
2260 GOSUB 770: GOSUB 20
20
2270 FOR I=1 TO NPLY
2280 H(I)=H(I)*S
2290 NEXT I
2300 S=0
2310 GOSUB 990: GOSUB 77
0: GOSUB 260
2320 RETURN
2330 '** TRANSFORM **
2340 FOR I=1 TO NPLY
2350 C2=COS(2*T(I)): C4=
COS(4*T(I))
2360 S2=SIN(2*T(I)): S4=
SIN(4*T(I))
2370 B(I,1)=U(1)+C2*U(2)
+C4*U(3)
2380 B(I,2)=U(1)-C2*U(2)
+C4*U(3)
2390 B(I,3)=U(4)-C4*U(3)
2400 B(I,4)=U(5)-C4*U(3)
2410 B(I,5)=S2/2*U(2)+S4
*U(3)
2420 B(I,6)=S2/2*U(2)-S4
*U(3)
2430 B(I,7)=U(6)+C2*U(7)
2440 B(I,8)=U(6)-C2*U(7)
2450 B(I,9)=S2*U(7)
2460 C(I,1)=U(1)+C2*U(2)
+C4*U(3)
2470 C(I,2)=U(1)-C2*U(2)
+C4*U(3)
2480 C(I,3)=U(4)-C4*U(3)
2490 C(I,4)=U(5)-C4*U(3)
2500 C(I,5)=S2/2*U(2)+S4
*U(3)
2510 C(I,6)=S2/2*U(2)-S4
*U(3)
2520 NEXT I
2530 RETURN
```

2030 - 2140 For each possible constraint solve for S in

$$G_{ij}\varepsilon_i\varepsilon_j \frac{(SREF)^2}{S^2} + G_i\varepsilon_i \frac{(SREF)}{S} - 1 = -E6$$

2150 Take smallest value (corresponds to closest constraint)

2200 - 2310 For equal ply ratios, find the smallest laminate thickness which does not violate any constraints. Initialize A matrix, strains, and constraint list

2370 - 2450 Transform failure parameters in following order

$B(I,1)=G_{11}$	$B(I,5)=G_{16}$
$B(I,2)=G_{22}$	$B(I,6)=G_{26}$
$B(I,3)=G_{12}$	$B(I,7)=G_1$
$B(I,4)=G_{66}$	$B(I,8)=G_2$
	$B(I,9)=G_3$

2460 - 2510 Transform modulus in following order

$C(I,1) = Q_{11}$	$C(I,5) = Q_{16}$
$C(I,L)=Q_{22}$	$C(I,6)=Q_{26}$
$C(I,3)=Q_{12}$	
$C(I,4)=Q_{66}$	

```
2540 '*** INPUT ***
2550 CLS
2600 PRINT "PRESS ANY KE
Y WHEN":PRINT "DESIRED M
ATERIAL":PRINT"APPEARS"
2610 FOR K=1 TO 750:NEXT
2620 FOR M=0 TO 6
2640 IF M=6 THEN M$="NEW
 MATERIAL" ELSE GET%M,EX
,EY,UX,ES,TPLY,XT,YT,XC,
YC,SS,M$
2650 CLS:PRINT M$:SOUND
20,1
2660 FOR J=1 TO 200
2670 IF INKEY$<>"" THEN
2700
2675 NEXT J,M
2680 GOTO 2620
2700 IF M=6 THEN GOSUB 9
000:GOTO 2600
2705 CLS:PRINT "▒ ";M$;"
▒"
2710 PRINT "HOW MANY"
2720 INPUT "PLY GROUPS";
NPLY
2730 CLS: PRINT "ENTER P
LY GROUP"
2740 PRINT "ORIENTATIONS
"
2750 FOR I=1 TO 200
2760 NEXT I
2770 CLS
2780 FOR I=1 TO NPLY
2790 PRINT "PLY ";I
2800 INPUT T(I)
2810 T(I)=FNRAD(T(I))
2820 NEXT I
2830 PRINT "ENTER NUMBER
 OF"
2840 PRINT "INDEPENDENT
LOAD"
2850 INPUT "CONDITIONS";
NL
2900 FOR I=1 TO NL
2910 CLS:PRINT "LOAD ";I
;" IN MPa."
2920 INPUT "N1=";XN(I,1)
2930 INPUT "N2=";XN(I,2)
2940 INPUT "N6=";XN(I,3)
2950 FOR J=1 TO 3
2960 XN(I,J)=XN(I,J)*1E6
2970 NEXT J,I
2980 RETURN
2990 '** INVARIENTS **
3050 VY=1/(1-UX*UX*EY/EX
)
3060 QXX=VY*EX*1E9: QYY=
VY*EY*1E9
3070 QXY=VY*UX*EY*1E9: Q
S=ES*1E9
3080 U(1)=(3*QXX+3*QYY+2
*QXY+4*QS)/8
3090 U(2)=(QXX-QYY)/2
3100 U(3)=(QXX+QYY-2*QXY
-4*QS)/8
```

2600 - 2675 List available materials. Get% is an HX-20 command to get data from a non-violatile RAM file

3050 - 3280 Calculate invariants for use in transformations. Note that some variables like EX and EY get reused, so their value may not be what you might expect after routine is called

```
3110 U(4)=(QXX+QYY+6*QXY
-4*QS)/8
3120 U(5)=(QXX+QYY-2*QXY
+4*QS)/8
3130 EX=1E-12/(XT*XC): E
Y=1E-12/(YT*YC): ES=1E-1
2/(SS*SS)
3140 FX=(1/XT-1/XC)/1E6:
 FY=(1/YT-1/YC)/1E6
3150 EXY=-SQR (EX*EY)/2
3160 GXX=EX*QXX*QXX+2*EX
Y*QXX*QXY+EY*QXY*QXY
3170 GYY=EX*QXY*QXY+2*EX
Y*QXY*QYY+EY*QYY*QYY
3180 GXY=EX*QXX*QXY+EXY*
(QXX*QYY+QXY*QXY)+EY*QXY
*QYY
3190 GSS=ES*QS*QS
3200 GX=FX*QXX+FY*QXY
3210 GY=FX*QXY+FY*QYY
3220 U(1)=(3*GXX+3*GYY+2
*GXY+4*GSS)/8
3230 U(2)=(GXX-GYY)/2
3240 U(3)=(GXX+GYY-2*GXY
-4*GSS)/8
3250 U(4)=(GXX+GYY+6*GXY
-4*GSS)/8
3260 U(5)=(GXX+GYY-2*GXY
+4*GSS)/8
3270 U(6)=(GX+GY)/2
3280 U(7)=(GX-GY)/2
3290 RETURN
3300 '** OUTPUT **
3302 SOUND 15,2:SOUND50,
2
3305 K$="Hit any key to
cont":U$="MN/m"
3310 CLS: TEST=0
3320 FOR I=1 TO NPLY
3330 TEST=TEST+H(I): NEX
T I
3350 PRINT "TOTAL THICKN
ESS="
3360 PRINT TEST;" m."
3370 PRINT USING "####.#
# Plies";TEST/TPLY
3375 PRINT K$;
3380 A$=INKEY$:IF A$<>""
 THEN 3380
3390 IF INKEY$="" THEN 3
390
3400 CLS:PRINT"Press Y i
f printout","of displaye
d result is  desired. Pr
ess Nif not";
3410 FOR I=1 TO 600:NEXT
 I
3415 A$=INKEY$:IF A$<>""
 THEN 3415
3420 CLS:RESTORE 6120
3425 JJ=0:A$=INKEY$
3430 FOR I=1 TO 8
3440 READ A$:CLS:PRINT:P
RINT A$:SOUND 20,1
3445 A$=INKEY$:IF A$=""
THEN 3445
3450 PRINT A$;:FOR KK=1
TO 75:NEXT KK
```

```
3455 IF A$="Y" THEN JJ=J
J+1:C%(JJ,1)=I
3460 NEXT I
3464 IF JJ<>0 THEN LPRIN
T STRING$(24,"▒")
3465 FOR KK=1 TO JJ
3470 ON C%(KK,1) GOSUB 5
000,5200,4000,4200,4400,
4600,4800,7500
3485 LPRINT
3490 NEXT KK
3495 CLS:PRINT"FINISHED"
,K$
3496 IF INKEY$=""THEN349
6 ELSE RUN
4000 '** PLY RATIO**
4002 CLS:LPRINT "Total t
hickness="
4004 LPRINT USING ".####
^^^^ m.";TEST
4006 LPRINT USING "####.
## Plies";TEST/TPLY
4008 LPRINT
4030 A$="ANGLE  RATIO  #
PLIES"
4040 LOCATE 0,1:PRINT A$
:LPRINT A$
4050 FOR I=1 TO NPLY
4060 A=CINT((FNDEG(T(I))
*1E2))/1E2
4070 B=CINT((H(I)/TEST*1
E4))/1E4
4080 C=CINT((H(I)/TPLY*1
E2))/1E2
4090 PRINT A;TAB(6);B;TA
B(13);C
4100 LPRINT A;TAB(6);B;T
AB(13);C
4120 NEXT I
4150 RETURN
4200 '** STRENGTH**
4210 LPRINT "STRENGTH RA
TIOS"
4215 LPRINT "1=ULTIMATE
STRAIN":
4220 LPRINT ">1 IS SAFE"
4225 FOR I=1 TO NL
4230 LPRINT "LOADING "I
4235 LPRINT "PLY"
4240 FOR P=1 TO NPLY
4245 IF H(P)=0 THEN 4305
4250 II=P:GOSUB 1230
4255 A#=0:B#=0
4260 FOR J=1 TO 3
4265 FOR K=1 TO 3
4270 A#=A#+G(J,K)*E(I,J)
*E(I,K)
4275 NEXT K
4280 B#=B#+S(J)*E(I,J)
4285 NEXT J
4290 A#=(-B#+SQR(B#*B#+4
*A#))/(2*A#)
4295 A=FIX(A#*1E4+.5)/1E
4
4300 LPRINT FNDEG(T(P));
TAB(10);A
4305 NEXT P,I
4310 RETURN
```

3400-3490 Branch for various output routines

4000-7560 Output routines and laminate analysis

4210 - 4305 Strength ratio is defined as the value of R in

$$G_{ij}\varepsilon_i\varepsilon_j R^2 + G_i\varepsilon_i R - 1 = 0$$

```
4400 '**STRAINS**
4410 LPRINT TAB(4);"LAMI
NATE STRAINS"
4420 FOR N=1 TO NL
4430 LPRINT "LOADING "N
4440 LPRINT USING "e1=+#
.###E-03";E(N,1)*1E3
4450 LPRINT USING "e2=+#
.###E-03";E(N,2)*1E3
4460 LPRINT USING "e6=+#
.###E-03";E(N,3)*1E3
4465 NEXT N
4470 RETURN
4600 ' **A MATRIX**
4610 CLS
4620 LPRINT"Norm. |A| in
 GPa."
4630 FOR I=1 TO 3
4640 FOR J=1 TO 3
4650 D(I,J)=A(I,J)/1E9/T
EST
4660 NEXT J,I
4670 GOSUB 7000
4680 RETURN
4800 'A INVERSE
4810 LPRINT"Compliance
(normalized)"
4820 LPRINT"in 1/TPa."
4830 FOR I=1 TO 3
4840 FOR J=1 TO 3
4850 D(I,J)=AI(I,J)*TEST
*1E12
4860 NEXT J,I
4870 GOSUB 7000
4880 RETURN
5000 LPRINT "Material Pr
operties"
5010 GET%M,EX,EY,UX,ES,T
PLY,XT,YT,XC,YC,SS,M$
5015 LPRINT M$
5020 LPRINT "EX=";EX;"GP
a"
5030 LPRINT "EY=";EY;"GP
a"
5040 LPRINT "ES=";ES;"GP
a"
5050 LPRINT "UX=";UX
5060 LPRINT "X=";XT;"MPa
"

5070 LPRINT "X'=";XC;"MP
a"
5072 LPRINT "Y=";YT;"MPa
"

5074 LPRINT "Y'=";YC;"MP
a"
5080 LPRINT "S=";SS;"MPa
"

5090 LPRINT "Ply Thickne
ss";TPLY"m"
5095 RETURN
5200 '**LOADS**
5210 FOR I=1 TO NL
5220 LPRINT "LOADING ";I
5230 FOR J=1 TO 3
5240 A$=STR$(J):IFJ=3THE
N A$=" 6"
5250 LPRINT "N";A$;"=";X
N(I,J)/1E6;U$
5260 NEXT J,I
5270 RETURN
```

```
6000 DATA 10,5E-5,.1,1E-
6
6120 DATA Ply properties
,Loads,Total thickness &
   ply ratios,Strength
ratios
6130 DATA Laminate strai
ns,Stiffness matrix,Comp
liance matrix,Engineerin
g const.
6500 '** ANALYSIS **
6505 CLS
6510 TEST=0
6520 FOR I=1 TO NPLY
6530 PRINT "NUMBER OF PL
IES AT",FNDEG(T(I));"DEG
REES":INPUT H(I)
6540 H(I)=H(I)*TPLY:TEST
=TEST+H(I)
6550 NEXT
6555 CLS
6560 S=0:GOSUB 2990:GOSU
B 2330:GOSUB 990:GOSUB 7
70
6570 GOTO 3300
7000 'FANCY
7010 LPRINT "┌──────┬──
─────┬─────┐"
7020 LPRINT USING "|###.
###";D(1,1),D(1,2),D(1,3
)
7030 A$="├──────┼──────
─┼─────┤"
7040 LPRINT A$
7050 LPRINT USING "|###.
###";D(2,1),D(2,2),D(2,3
)
7060 LPRINT A$
7070 LPRINT USING "|###.
###";D(3,1),D(3,2),D(3,3
)
7080 LPRINT "└──────┴──
─────┴─────┘"
7100 RETURN
7500 '** ENG. CONST. **
7505 LPRINT "ENGINEERING
 CONSTANTS":LPRINT
7510 LPRINT USING"E1=###
.# GPa";1/AI(1,1)/TEST/1
E9
7515 LPRINT USING"E2=###
.# GPa";1/AI(2,2)/TEST/1
E9
7520 LPRINT USING"E6=###
.# GPa";1/AI(3,3)/TEST/1
E9
7525 LPRINT USING"E2=###
.# GPa";1/AI(2,2)/TEST/1
E9
7530 LPRINT USING"v21=##
.###";-AI(1,2)/AI(1,1)
7540 LPRINT USING"v61=##
.###";AI(1,3)/AI(1,1)
7550 LPRINT USING"v16=##
.###";AI(1,3)/AI(3,3)
7560 RETURN
```

6500-6570 MAIN for performing only laminate analysis (number of plies is a given)

```
9000 PRINT"REVIEW OR NEW
"
9010 INPUT "DATA (R/N)";
A$
9020 IF A$="R" THEN 9190
9030 PRINT"WHICH MATERIA
L":PRINT"WILL YOU"
9040 INPUT "REPLACE (0-5
)";I
9050 INPUT "EX(GPa)=";EX
9060 INPUT "EY(GPa)=";EY
9070 INPUT "VX=";VX
9075 INPUT "ES(GPa)=";ES
9080 INPUT "X(MPa)=";X
9090 INPUT "X'(MPa)=";XX
9100 INPUT "Y(MPa)=";Y
9110 INPUT "Y'(MPa)=";YY
9120 INPUT "S(MPa)=";S
9130 INPUT "THICKNESS (m
)=";TPLY
9140 INPUT "NAME (15 CHR
. MAX.)";M$
9150 PUT%I,EX,EY,VX,ES,T
PLY,X,Y,XX,YY,S,M$
9160 PRINT "ADDITIONAL":
INPUT "CHANGES (Y/N)";A$
9170 IF A$="Y" THEN 9000
9180 RETURN
9190 PRINT"REVIEW WHICH"
:INPUT"MATERIAL (0-5)";M
9200 GOSUB5000
9210 GOTO 9160
9500 OPEN "I",#1,"CAS1:D
ATA"
9510 FOR I=0 TO 5
9520 INPUT #1,EX,EY,VX,E
S,T,X,Y,XX,YY,S,M$
9530 PUT%I,EX,EY,VX,ES,T
,X,Y,XX,YY,S,M$
9540 NEXT
9550 CLOSE #1
9560 DELETE 45
9570 GOTO 50
10000 'THETA
10010 L=0
10020 SREF=0
10030 FOR I=1 TO NPLY
10040 SREF=SREF+H(I)*H(I
)
10050 NEXT
10060 FOR P=1 TO NPLY
10070 C2=2*COS(2*T(P))
10080 C4=4*COS(4*T(P))
10090 S2=2*SIN(2*T(P))
10100 S4=4*SIN(4*T(P))
10110 D(1,1)=-V(2)*S2-V(
3)*S4
10120 D(2,2)=V(2)*S2-V(3
)*S4
10130 D(3,3)=V(3)*S4
10140 D(1,3)=V(2)*C2/2+V
(3)*C4
10150 D(2,3)=V(2)*C2/2-V
(3)*C4
```

9000-9210 Enter material properties into library

9500-9570 Routine for automatically entering material properties from a cassette tape. To use, line 45 should read GOTO 9500, and load program. With tape still connected, run program and the properties will load.

10000 Angle optimization subroutine

10110-10190 Angular derivatives of failure parameters

```
10160 D(1,2)=D(3,3):D(2,
1)=D(1,2):D(3,2)=D(2,3):
D(3,1)=D(1,3)
10170 X(1)=-U(7)*S2
10180 X(2)=-X(1)
10190 X(3)=U(7)*C2
10200 II=P:GOSUB 1230
10210 FOR N=1 TO NL
10220 FCON=-1
10230 FOR K=1 TO 3
10240 FOR J=1 TO 3
10250 FCON=FCON+G(J,K)*E
(N,J)*E(N,K)
10260 NEXT J
10270 FCON=FCON+S(K)*E(N
,K)
10280 NEXT K
10290 L=L+1
10300 CON(L)=FCON
10310 FOR PP=1 TO NPLY
10320 C2=2*COS(2*T(PP)):
C4=4*COS(4*T(PP))
10330 S2=2*SIN(2*T(PP)):
S4=4*SIN(4*T(PP))
10340 A(1,1)=-U(2)*S2-U(
3)*S4
10350 A(2,2)=U(2)*S2-U(3
)*S4
10360 A(3,3)=U(3)*S4
10370 A(1,3)=U(2)*C2/2+U
(3)*C4
10380 A(2,3)=U(2)*C2/2-U
(3)*C4
10390 A(1,2)=A(3,3):A(2,
1)=A(1,2):A(3,2)=A(2,3):
A(3,1)=A(1,3)
10400 FOR J=1 TO 3
10410 R(J)=0
10420 FOR K=1 TO 3
10430 R(J)=R(J)+A(J,K)*E
(N,K)*H(P)
10440 NEXT K,J
10450 FOR J=1 TO 3
10460 Y(J)=0
10470 FOR K=1 TO 3
10480 Y(J)=Y(J)-AI(J,K)*
R(K)
10490 NEXT K,J
10500 DUM=0
10510 FOR J=1 TO 3
10520 FOR K=1 TO 3
10530 DUM=DUM+G(J,K)*(Y(
J)*E(N,K)+E(N,J)*Y(K))
10540 IF P=PP THEN DUM=D
UM+D(J,K)*E(N,J)*E(N,K)
10550 NEXT K
10560 DUM=DUM+S(J)*Y(J)
10570 IF P=PP THEN DUM=D
UM+X(J)*E(N,J)
10580 NEXT J
10590 W(L,PP)=DUM
10600 NEXT PP
10610 NEXT N,P
10620 ZMAX=0
10630 FOR P=1 TO NPLY
10640 DUM=0:DUM2=0:DUM3=
0
```

10250-10270 Calculate value of failure equation for each load

10340-10390 Angular derivatives of Q matrix terms

10400-10490 Solve for partials of strain with respect to angle

10510-10600 Solve equation (29)

10630-10700 Sove for equation (28)

```
10650 FOR J=1 TO L
10660 DUM=DUM+CON(J)*W(J
,P)
10670 DUM2=DUM2+W(J,P)
10680 DUM3=DUM3+CON(J)
10690 NEXT J
10700 Z(P)=DUM-DUM2*DUM3
/NPLY
10710 IF ABS(Z(P))>ZMAX
AND Z(P)<>0 THEN ZMAX=AB
S(Z(P))
10720 X(P)=0
10730 NEXT P
10740 FOR I=1 TO NPLY
10750 IF ZMAX=0 THEN RET
URN ELSE Z(I)=-Z(I)/ZMAX
10760 NEXT I
10770 T=1:TEST=SREF
10780 FOR I=1 TO NPLY
10790 CON(I)=X(I):X(I)=Z
(I)*T
10800 X(I)=CINT(X(I))
10810 T(I)=T(I)+(X(I)-CO
N(I))*DELTA
10820 NEXT
10830 GOSUB 2330:S=0:GOS
UB 990:GOSUB 770
10840 GOSUB 2020
10850 IF S<TEST THEN T=T
+1:TEST=S:GOTO 10780
10860 FOR I=1 TO NPLY
10870 T(I)=T(I)-(X(I)-CO
N(I))*DELTA
10880 NEXT
10890 S=0:GOSUB 2330:GOS
UB 990:GOSUB 770:GOSUB 2
020
10900 FOR I=1 TO NPLY
10910 H(I)=H(I)*S/SREF
10930 II=P:GOSUB 1230
10940 NEXT
10960 IF T=1 THEN RETURN
 ELSE GOTO 10000
```

10740-10760 Normalize Z by largest component

10780-10820 Incremental step of all angles

10830 Update transformations, strains, and scale total thickness

10860-10880 After minimum past, go back one step

10900-10940 Update ply group thickness

10960 If any progress made, go back and try a new direction. If not, return to main

APPENDIX C

Orthotropic Laminate Optimization Program

The following program produces a thickness optimized laminate that is constraint to be orthotropic. A search can be made for the best orientation of the orthotropic axis. In the final result, ply angles are measured from one of the orthotropic axes (the original 1 axis) plus a rigid body rotation is given. Angles appear to stay constant, but the rigid body rotation must be added to get the angle to the laminate 1 axis (see Figure 15). The failure theory is based on the strain sphere approximation of the first-ply failure inner-envelope. The laminate must remain balanced. Instead of entering both the plus and minus angle, only one is entered and the program assumes the presence of the other. The final thickness must be divided evenly between a plus theta and a minus theta ply group.

Running the program is similar to running the program listed in Appendix B. The only differences are that if optimimum orientation is desired, the search limits and maximum error must be entered. The search limits are the angles between which the best laminate is thought to lie. If a minimum is not found between the given limits, the program automatically extends the limits, but this is time consuming.
All angles (and the error) are entered as degrees.

```
10 '** MAIN CLASS**
20 WIDTH 20,10
30 CLEAR 75,330
40 DEFFIL 55,0
50 DIM XN(3,3),Q11(4),Q2
2(4),Q12(4),Q66(4),H(4),
R(3),T(4)
60 DIM X(4),Y(3),Z(4),E(
3,3),C$(4),U(5),V(5)
70 DEF FNRAD(X)=X/180*3.
14159
80 DEF FNDEG(X)=X*180/3.
14159
90 RESTORE
100 READ E2,E5,E6
110 ITER=1
120 GOSUB 1830
130 GOSUB 2180
140 GOSUB 1720
150 INPUT "OPT. ROTATION
 (Y/N)";A$
160 IF A$="Y" THEN GOTO
4180
170 GOSUB 1580
180 GOSUB 1160
190 CLS:PRINT "WORKING,I
TERATION";ITER
200 IF F$="FAIL" THEN 25
0
210 GOSUB 840
220 ITER=ITER+1
230 IF F$="FAIL" THEN 25
0
240 GOTO 180
250 H=0
260 FOR I=1 TO NPLY
270 H=H+H(I):NEXT I
280 GOTO 2330
290 '** CONSTRAINT TEST*
*
300 G$="PASS": NC=0
310 FOR N=1 TO NL
320 FCON=(E(N,1)*E(N,1)+
E(N,2)*E(N,2)+E(N,3)*E(N
,3)/2)/EMAX-1
330 IF FCON>0 THEN G$="F
AIL":RETURN
340 IF FCON<-E5 THEN GOT
O 370
350 NC=NC+1
360 C$(NC)=CHR$(N)
370 NEXT N
380 RETURN
400 '** GRADIENT **
410 UNORM=0
420 FOR L=1 TO NPLY
430 IF H(L)=0 THEN Z(L)=
0:GOTO 520
440 R(1)=-Q11(L)*E(N,1)-
Q12(L)*E(N,2)
450 R(2)=-Q12(L)*E(N,1)-
Q22(L)*E(N,2)
460 R(3)=-Q66(L)*E(N,3)
470 Y(1)=AI11*R(1)+AI12*
R(2)
480 Y(2)=AI12*R(1)+AI22*
R(2)
```

100 Error and numerical offset constants

120 Input

130 Invarients

140 Transformations

160 Branch for optimum orientation

170 Initial feasible point

180 Direction

210 New position in design space

300-380 Equation (9)

400-520 Partial derivatives of strain

```
490 Y(3)=AI66*R(3)
500 Z(L)=2*Y(1)*E(N,1)+2
*Y(2)*E(N,2)+Y(3)*E(N,3)
510 UNORM=UNORM+Z(L)*Z(L
)
520 NEXT L
530 UNORM=SQR(UNORM)
540 FOR L=1 TO NPLY
550 Z(L)=Z(L)/UNORM
560 NEXT L
570 RETURN
580 '** STRAINS **
590 F11=A11+D11*S
600 F12=A12+D12*S
610 F22=A22+D22*S
620 F66=A66+D66*S
630 DET=F11*F22-F12*F12
640 AI11=F22/DET
650 AI22=F11/DET
660 AI12=-F12/DET
670 AI66=1/F66
680 FOR I=1 TO NL
690 E(I,1)=AI11*XN(I,1)+
AI12*XN(I,2)
700 E(I,2)=AI12*XN(I,1)+
AI22*XN(I,2)
710 E(I,3)=AI66*XN(I,3)
720 NEXT I
730 RETURN
740 '** A MATRIX **
750 A11=0:A22=0:A12=0:A6
6=0
760 D11=0:D22=0:D12=0:D6
6=0
770 FOR I=1 TO NPLY
780 A11=A11+Q11(I)*H(I):
D11=D11+Q11(I)*Z(I)
790 A22=A22+Q22(I)*H(I):
D22=D22+Q22(I)*Z(I)
800 A12=A12+Q12(I)*H(I):
D12=D12+Q12(I)*Z(I)
810 A66=A66 +Q66(I)*H(I)
:D66=D66+Q66(I)*Z(I)
820 NEXT I
830 RETURN
840 REM ***NEW POSITION
***
850 SMAX=1E10
860 FOR I=1 TO NPLY
870 IF Z(I)<>0 THEN S=-H
(I)/Z(I)
880 IF S>0 AND S<SMAX TH
EN SMAX=S
890 NEXT I
900 F$=""
910 IF SMAX> 10 THEN F$=
"FAIL": RETURN
920 S1=0: S2=SMAX: S=SMA
X
930 IF NC=0 THEN 1070
940 GOSUB 580: GOSUB 290
950 IF G$="FAIL" THEN S2
=S ELSE S1=S
960 IF S1=SMAX THEN 1010
970 S=(S1+S2)/2
980 IF S2-S1<E2 AND S1=0
 THEN F$="FAIL": S=0: GO
TO 1130
990 IF S1/(S2-S1)<4 THEN
 940
1000 S=S/2
```

530-560 Normalize gradient

590-620 Update A matrix for point S

600-670 Invert A assuming orthotropic laminate

680-710 Solve for strains

750-800 Form A matrix

850-1000 Bisection search for next constraint

```
1010 SREF=0
1020 FOR I=1 TO NPLY
1030 H(I)=H(I)+Z(I)*S
1040 IF H(I)<E2 THEN H(I
)=0
1050 SREF=SREF+H(I)*H(I)
1060 NEXT I
1070 S=0:SREF=SQR(SREF)
1080 GOSUB 740: GOSUB 58
0: GOSUB 1510
1090 IF SREF-S<E2 THEN F
$="FAIL"
1100 FOR I=1 TO NPLY
1110 H(I)=H(I)*S/SREF
1120 NEXT I
1130 S=0
1140 GOSUB 740: GOSUB 58
0: GOSUB 290
1150 RETURN
1160 '** DIRECTION **
1170 W=0: UNORM=1
1180 FOR I=1 TO NPLY
1190 X(I)=0
1200 W=W+SGN(H(I))
1210 NEXT I
1220 W=1/SQR(W)
1230 IF NC=0 THEN 1350
1240 FOR I=1 TO NC
1250 N=ASC(C$(I))
1260 GOSUB 400
1270 FOR J=1 TO NPLY
1280 LET X(J)=X(J)-Z(J)
1290 NEXT J,I
1300 UNORM=0
1310 FOR J=1 TO NPLY
1320 UNORM=UNORM+X(J)*X(
J)
1330 NEXT J
1340 UNORM=SQR(UNORM): T
EST=0
1350 FOR I=1 TO NPLY
1360 X(I)=X(I)/UNORM
1370 TEST=TEST+X(I)*W*SG
N(H(I))
1380 NEXT I
1390 UNORM=0
1400 FOR I=1 TO NPLY
1410 Z(I)=X(I)-TEST*W*SG
N(H(I))
1420 UNORM=UNORM+Z(I)*Z(
I)
1430 NEXT I
1440 IF UNORM<1E-6 THEN
F$="FAIL": RETURN  ELSE
F$=""
1450 UNORM=SQR(UNORM)
1460 FOR I=1 TO NPLY
1470 Z(I)=Z(I)/UNORM
1480 NEXT I
1490 GOSUB 740
1500 RETURN
1510 '** STRAIN RATIO **
1520 FOR N=1 TO NL
1530 SVAL=SREF*SREF/(1-E
6)/EMAX*(E(N,1)*E(N,1)+E
(N,2)*E(N,2)+E(N,3)*E(N,
3)/2)
```

1010-1140 At halfway point, rescale lamiate and update thickness vector

1170-1260 Get gradient of each active constraint

1270-1340 Sum gradients and normalize result

1350-1490 Project onto constant thickness plane. Test for minimum and normalize final result

1520-1560 Find distance from farthest constraint to origin

```
1540 SVAL=SQR(SVAL)
1550 IF SVAL>S THEN S=SV
AL
1560 NEXT N
1570 RETURN
1580 '** IFP **
1590 W=1/SQR(NPLY)
1600 FOR I=1 TO NPLY
1610 Z(I)=W: H(I)=W
1620 NEXT I
1630 GOSUB 740
1640 S=0: SKEF=1
1650 GOSUB 580: GOSUB 15
10
1660 FOR I=1 TO NPLY
1670 H(I)=H(I)*S
1680 NEXT I
1690 S=0
1700 GOSUB 740: GOSUB 58
0: GOSUB 290
1710 RETURN
1720 '** TRANSFORM **
1730 J=QXX+QYY+2*QXY:K=Q
SS-QXY
1740 FOR I=1 TO NPLY
1750 C2=COS(T(I)):C2=C2*
C2
1760 S2=SIN(T(I)):S2=S2*
S2
1770 Q11(I)=C2*C2*QXX+S2
*S2*QYY+2*S2*C2*(QXY+2*Q
SS)
1780 Q22(I)=S2*S2*QXX+C2
*C2*QYY+2*C2*S2*(QXY+2*Q
SS)
1790 Q12(I)=(J-(Q11(I)+Q
22(I)))/2
1800 Q66(I)=(J+2*K-(Q11(
I)+Q22(I)))/2
1810 NEXT I
1820 RETURN
1830 '*** INPUT ***
1840 CLS
1850 PRINT "PRESS ANY KE
Y WHEN ","DESIRED MATERI
AL","APPEARS"
1860 FOR K=1 TO 750:NEXT
1870 FOR M=0 TO 6
1880 IF M=6 THEN M$="NEW
 MATERIAL" ELSE GET%M,EX
,EY,VX,ES,TPLY,XT,YT,XC,
YC,SS,M$
1890 CLS:PRINT M$:SOUND
20,1
1900 FOR J=1 TO 200
1910 IF INKEY$<>"" THEN
1940
1920 NEXT J,M
1930 GOTO 1850
1940 IF M=6 THEN GOSUB 3
750:GOTO 1850
1950 CLS:PRINT "▒ ";M$;"
▒"
1960 PRINT "HOW MANY"
1970 INPUT "PLY GROUPS";
NPLY
1980 CLS:PRINT"ENTER PLY
 GROUP"
```

1730-1810 Transformation of elasticity matrix, assuming orthotropic laminate

```
2470 CLS:PRINT"Press Y i
f printout","of displaye
d result is desired. Pre
ss N","if not";
2480 ITER=1
2490 FOR I=1 TO 600:NEXT
 I
2500 A$=INKEY$:IF A$<>""
 THEN 2500
2510 CLS:RESTORE 4630
2520 J=0:A$=INKEY$
2530 FOR I=1 TO 8
2540 READ A$:CLS:PRINT:P
RINT A$:SOUND 20,1
2550 A$=INKEY$:IF A$=""
THEN 2550
2560 PRINT A$;:FOR KK=1
TO 75:NEXT KK
2570 IF A$="Y" THEN J=J+
1:C%(J,1)=I
2580 NEXT I
2590 FOR K=1 TO J
2600 ON C%(K,1) GOSUB 32
10,3350,2650,2780,2890,2
980,3090
2610 LPRINT
2620 NEXT K
2630 CLS:PRINT"FINISHED"
,K$
2640 IF INKEY$=""THEN264
0 ELSE RUN
2650 '** PLY RATIO**
2660 CLS:LPRINT "Total t
hickness="
2670 LPRINT USING ".####
^^^^ m.";TEST
2680 LPRINT USING "####.
## Plies";TEST/TPLY
2690 LPRINT
2700 LPRINT "ANGLE  RATI
O  #PLIES"
2710 FOR I=1 TO NPLY
2720 A=CINT((FNDEG(T(I))
*1E2))/1E2
2730 B=CINT((H(I)/TEST*1
E4))/1E4
2740 C=CINT((H(I)/TPLY*1
E2))/1E2
2750 LPRINT A;TAB(6);B;T
AB(13);C
2760 NEXT I
2770 RETURN
2780 '** STRENGTH**
2790 LPRINT "STRENGTH RA
TIOS"
2800 LPRINT "1=ULTIMATE
STRAIN":
2810 LPRINT ">1 IS SAFE"
2820 FOR I=1 TO NL
2830 LPRINT "LOADING "I
2840 A=E(I,1)*E(I,1)+E(I
,2)*E(I,2)-E(I,3)*E(I,3)
/2
2850 A=SQR(EMAX/A)
2860 LPRINT "R=";A
2870 NEXT
2880 RETURN
```

2510-2620 Branch for various output routines

2650-3520 Output routines and laminate analysis

```
2890 '**STRAINS**
2900 LPRINT TAB(4);"LAMI
NATE STRAINS"
2910 FOR N=1 TO NL
2920 LPRINT "LOADING "N
2930 LPRINT USING "e1=+#
.###E-03";E(N,1)*1E3
2940 LPRINT USING "e2=+#
.###E-03";E(N,2)*1E3
2950 LPRINT USING "e6=+#
.###E-03";E(N,3)*1E3
2960 NEXT N
2970 RETURN
2980 ' **A MATRIX**
2990 CLS
3000 LPRINT"Norm. |A| in
 GPa."
3010 D(1,1)=A11:D(1,2)=A
12:D(2,2)=A22:D(3,3)=A66
3020 D(1,3)=0:D(3,1)=0:D
(2,3)=0:D(3,2)=0:D(2,1)=
D(1,2)
3030 FOR I=1 TO 3
3040 FOR J=1 TO 3
3050 D(I,J)=D(I,J)/TEST/
1E9
3060 NEXT J,I
3070 GOSUB 3430
3080 RETURN
3090 'A INVERSE
3100 LPRINT"Compliance
(normalized)"
3110 LPRINT"in 1/TPa."
3120 D(1,1)=AI11:D(1,2)=
AI12:D(2,2)=AI22:D(3,3)=
AI66
3130 D(1,3)=0:D(2,3)=0:D
(3,2)=0:D(3,1)=0
3140 D(2,1)=D(1,2)
3150 FOR I=1 TO 3
3160 FOR J=1 TO 3
3170 D(I,J)=D(I,J)*TEST*
1E12
3180 NEXT J,I
3190 GOSUB 3430
3200 RETURN
3210 LPRINT "Material Pr
operties"
3220 GET%M,EX,EY,UX,ES,T
PLY,XT,YT,XC,YC,SS,M$
3230 LPRINT M$
3240 LPRINT "EX=";EX;"GP
a"
3250 LPRINT "EY=";EY;"GP
a"
3260 LPRINT "ES=";ES;"GP
a"
3270 LPRINT "UX=";UX
3280 LPRINT "X=";XT;"MPa
"
3290 LPRINT "X'=";XC;"MP
a"
3300 LPRINT "Y=";YT;"MPa
"
3310 LPRINT "Y'=";YC;"MP
a"
3320 LPRINT "S=";SS;"MPa
"
3330 LPRINT "Ply Thickne
ss";TPLY"m"
3340 RETURN
```

```
3350 '**LOADS**
3360 FOR I=1 TO NL
3370 LPRINT "LOADING ";I
3380 FOR J=1 TO 3
3390 A$=STR$(J):IFJ=3THE
N A$=" 6"
3395 A=CINT(XN(I,J)/1E3)
/1E3
3400 LPRINT "N";A$;"=";A
;U$
3410 NEXT J,I
3420 RETURN
3430 'FANCY
3440 LPRINT "┌───────┬───
──────┬──────┐"
3450 LPRINT USING "|###.
###";D(1,1),D(1,2),D(1,3
)
3460 A$="├───────┼──────
─┼──────┤"
3470 LPRINT A$
3480 LPRINT USING "|###.
###";D(2,1),D(2,2),D(2,3
)
3490 LPRINT A$
3500 LPRINT USING "|###.
###";D(3,1),D(3,2),D(3,3
)
3510 LPRINT "└───────┴───
──────┴──────┘"
3520 RETURN
```

```
3750 PRINT"REVIEW OR NEW
"
3760 INPUT "DATA (R/N)";
A$
3770 IF A$="R" THEN 3950
3780 PRINT"WHICH MATERIA
L":PRINT"WILL YOU"
3790 INPUT "REPLACE (0-5
)";I
3800 INPUT "EX(GPa)=";EX
3810 INPUT "EY(GPa)=";EY
3820 INPUT "UX=";UX
3830 INPUT "ES(GPa)=";ES
3840 INPUT "X(MPa)=";X
3850 INPUT "X'(MPa)=";XX
3860 INPUT "Y(MPa)=";Y
3870 INPUT "Y'(MPa)=";YY
3880 INPUT "S(MPa)=";S
3890 INPUT "THICKNESS (m
)=";TPLY
3900 INPUT "NAME (15 CHR
. MAX.)";M$
3910 PUT%I,EX,EY,UX,ES,T
PLY,X,Y,XX,YY,S,M$
3920 PRINT "ADDITIONAL":
INPUT "CHANGES (Y/N)";A$
3930 IF A$="Y" THEN 3750
3940 RETURN
3950 PRINT"REVIEW WHICH"
:INPUT"MATERIAL (0-5)";M
3960 GOSUB3210
3970 GOTO 3920
```

```
3980 OPEN "I",#1,"CAS1:D
ATA"
3990 FOR I=0 TO 5
4000 INPUT #1,EX,EY,UX,E
S,T,X,Y,XX,YY,S,M$
4010 PUT%I,EX,EY,UX,ES,T
,X,Y,XX,YY,S,M$
4020 NEXT
4030 CLOSE #1
4040 DELETE 45
4060 '***ROTATION***
4065 CLS:PRINT "WORKING"
4070 A=-ROT-ROTSUM:ROTSU
M=ROTSUM+A
4080 A=FNRAD(-A)
4090 C=COS(A):C2=C*C
4100 S=SIN(A):S2=S*S
4110 FOR I=1 TO NL
4120 R(1)=XN(I,1)*C2+XN(
I,2)*S2+XN(I,3)*2*S*C
4130 R(2)=XN(I,1)*S2+XN(
I,2)*C2-XN(I,3)*2*S*C
4140 R(3)=-XN(I,1)*C*S+X
N(I,2)*C*S+XN(I,3)*(C2-S
2)
4150 XN(I,1)=R(1):XN(I,2
)=R(2):XN(I,3)=R(3)
4160 NEXT I
4170 RETURN
4180 '**ANGLE SEARCH**
4190 INPUT "LOWER SEARCH
 LIMIT";S1
4195 INPUT "UPPER SEARCH
 LIMIT";S2
4200 IF S2<=S1 THEN 4190
4210 INPUT "MAX. ERROR";
E1
4220 ROTSUM=0
4230 U(1)=S1:U(5)=S2
4240 U(3)=(S1+S2)/2
4250 U(2)=(U(3)+U(1))/2
4260 U(4)=(U(5)+U(3))/2
4265 AA=1:BB=5
4270 FOR II=AA TO BB
4280 ROT=U(II)
4290 GOSUB 4060:GOSUB 46
50
4300 V(II)=H
4310 NEXT
4320 VMIN=V(1):J=1
4330 FOR I=2 TO 5
4340 IF V(I)<VMIN THEN V
MIN=V(I):J=I
4350 NEXT
4360 IF J=1 THEN 4820
4365 IF J=5 THEN 4770
4370 Y(1)=V(J-1):X(1)=U(
J-1)
4380 Y(2)=V(J):X(2)=U(J)
4390 Y(3)=V(J+1):X(3)=U(
J+1)
4400 U(1)=X(1):U(3)=X(2)
:U(5)=X(3)
4410 V(1)=Y(1):V(3)=Y(2)
:V(5)=Y(3)
4420 IF U(5)-U(1)<=E1 TH
EN 4540
```

3980-4040 Routine for automatically entering material properties from cassette tape. To use, line 45 should read GOTO 3980, and run program with tape still connected

4070-4170 Transform loads to new axis system.

4220-4610 One-dimensional search for best rigid body rotation

```
1990 PRINT "ORIENTATIONS
"
2000 FOR I=1 TO 350
2010 NEXT
2020 CLS
2030 FOR I=1 TO NPLY
2040 PRINT "PLY ";I
2050 INPUT T(I)
2060 T(I)=FNRAD(T(I))
2070 NEXT I
2080 INPUT "NUMBER OF LO
ADINGS=";NL
2090 FOR I=1 TO NL
2100 CLS:PRINT "LOADING
";I
2110 INPUT "N1(MPa)=";XN
(I,1)
2120 INPUT "N2(MPa)=";XN
(I,2)
2130 INPUT "N6(MPa)=";XN
(I,3)
2140 FOR J=1 TO 3
2150 XN(I,J)=XN(I,J)*1E6
2160 NEXT J:NEXT I
2170 RETURN
2180 '** INVARIENTS**
2190 VY=1/(1-VX*VX*EY/EX
)
2200 QXX=VY*EX*1E9:QYY=V
Y*EY*1E9
2210 QXY=VY*VX*EY*1E9:QS
S=ES*1E9
```

2190-2210 Calculate Q's from engineering constants

```
2220 U(1)=XT/EX
2230 U(2)=XC/EX
2240 U(3)=YT/EY
2250 U(4)=YC/EY
2260 U(5)=SS/ES/SQR(2)
2270 EMAX=U(1)*U(1)/1E6
2280 FOR I=2 TO 5
2290 U(I)=U(I)*U(I)/1E6
2300 IF U(I)<EMAX THEN E
MAX=U(I)
2310 NEXT I
```

2220-2310 Find smallest strain component. It becomes the radius of the strain-sphere

```
2320 RETURN
2330 '** OUTPUT **
2340 SOUND 15,2:SOUND50,
2
2350 K$="Hit any key to
cont.":U$="MN/m"
2360 CLS:TEST=0
2365 IF F$="ROT" THEN LP
RINT "RIGID BODY","ROTAT
ION OF";U(3),"DEGREES"
2370 FOR I=1 TO NPLY
2380 TEST=TEST+H(I):NEXT
 I
2390 PRINT "TOTAL THICKN
ESS="
2400 PRINT TEST;" m."
2410 PRINT USING "####.#
# Plies";TEST/TPLY
2420 PRINT K$;
2430 LOCATE 0,0
2440 A$=INKEY$:IF A$<>""
 THEN 2440
2450 IF INKEY$="" THEN 2
450
```

```
4430 U(2)=(U(3)+U(1))/2
4440 U(4)=(U(5)+U(3))/2
4450 FOR II=2 TO 4 STEP
2
4460 ROT=U(II)
4470 GOSUB 4060:GOSUB 46
50
4480 U(II)=H
4490 NEXT
4500 GOTO 4320
4540 ROT=U(3)
4550 GOSUB 4060:GOSUB 46
50
4560 ROT=0:GOSUB 4060
4570 PRINT "OPT. ORIENTA
TION=";U(3)
4580 PRINT "TOTAL OF ";U
(3)/TPLY,"PLIES";
4590 SOUND 15,2:SOUND 30
,2
4600 IF INKEY$="" THEN 4
600
4605 F$="ROT"
4610 GOTO 2330
4620 DATA 5E-5,.1,1E-6
4630 DATA Ply properties
,Loads,Total thickness &
    ply ratios,Strength
ratios
4640 DATA Laminate strai
ns,Stiffness matrix,Comp
liance matrix

4650 '**OPT. RATIO**
4660 GOSUB 1530
4670 GOSUB 1160
4680 IF F$="FAIL" THEN 4
720
4690 GOSUB 840
4700 IF F$="FAIL" THEN 4
720
4710 GOTO 4670
4720 H=0
4730 FOR I=1 TO NPLY
4740 H=H+H(I)
4750 NEXT I
4760 RETURN
4770 U(1)=U(4):U(1)=U(4)
:U(2)=U(5):U(2)=U(5)
4780 DEL=U(2)-U(1)
4790 U(3)=U(2)+DEL:U(4)=
U(3)+DEL:U(5)=U(4)+DEL
4800 AA=3:BB=5
4810 GOTO 4270
4820 U(5)=U(2):U(5)=U(2)
:U(4)=U(1):U(4)=U(1)
4830 DEL=U(5)-U(4)
4840 U(3)=U(4)-DEL:U(2)=
U(3)-DEL:U(1)=U(2)-DEL
4850 AA=1:BB=3
4860 GOTO 4270
```

4650-4760 Version of MAIN used by optimum angle search to optimize ply ratios at each trial orientation

4770-4860 Routine to adjust search bounds if minimum not found in given limits

VITA

Gerald V. Flanagan was born in Luray, Virginia in 1956. He attended the Massachuetts Institute of Technology were he obtained an S.B. in Aeronautical and Astronautical Engineering, and recieved his Air Force commission as a Second Lieutenant through AFROTC. While at MIT he worked as a research assistant at the Technology Laboratory for Advanced Composites under Dr. James Mar. From 1979-1982 he worked at the Air Force Foriegn Technology Division as a propulsion system technology analyst. This was followed by an assignment to the Air Force Materials Laboratory's Mechanics and Surface Interactions Branch under Dr. Steven Tsai. There, his research responsibilities included design optimization and mechanics of composites.

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The design of a composite panel requires some way of finding the minimum thickness laminate which will withstand the load requirements without failure. The mathematical complexity of this problem dictates the use of non-linear optimization techniques. Although there are sophisticated optimization programs available capable of solving for the ply ratios, these programs are not often used in preliminary design because they require a large computer and some knowledge of the program's operation. As an alternative, specialized laminate

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optimization programs were developed which are compact and efficient enough to run on microcomputers. Only stresses at a point and inplane loads and deflections are considered. The programs are simple to use and require no knowledge of optimization. Techniques are developed in this thesis that find minimum thickness laminates with either ply ratios or ply angles as design variables. In addition, a method is presented for finding the optimum orientation for the axis of symmetry of an orthotropic laminate. The orthotropic laminate program uses an approximate failure theory, as suggested by Tsai, that has been found to speed computations dramatically.

Many test cases were run with these programs to demonstrate the weight savings possible over quasi-isotropic laminates. Of particular interest is performance of the laminates under multiple independent loads. Initial orientations for the programs to operate on were studied, and 0/90/45/-45 laminates were found to be an effective starting point for design.

The approximate failure criterion made analytic investigations of optimized laminates possible. A method of plotting maximum strain energy density as a function of the shear-stress-free laminate orientation is derived to demonstrate how the laminates adapt to multiple design load requirements in the optimization process. Also, an optimality criterion is derived which is satisfied by each ply group at the minimum thickness condition.

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